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ABSTRACT

This monograph is the third in a series of three dealing with the curriculum and evaluation standards for grades K-4 as promulgated by the National Council of Teachers of Mathematics (NCTM). The publication of the standards marks a new guidepost in the evolution of mathematics education and signals the impact of the information society upon all citizens through the recognition of the importance of the mathematical skills involving problem solving, estimation, reasonableness of computed results, and use of technology among others. Included in this third volume are: (1) introductory information concerning various perspectives on the teaching and learning of elementary mathematics; (2) essays, sample problems, and activities dealing with Standard #10, measurement; (3) activities revolving around Standard #11, statistics and probability; (4) sample problems and activities concerning Standard #12, fractions and decimals; (5) reflections and activities involving Standard #13, patterns and relationships; (6) additional activities designed for the enrichment of mathematical thought; and (7) appendices that contain ideas for alternative evaluation methods and for practical uses of the NCTM publication, "The Arithmetic Teacher." (JJK)

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Monograph No. 3 Spring 1990 Standards 10 through 13
A publication of the Virginia Council of Teachers of Mathematics

Implementing the Mathematics Standards

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A monograph for all who are interested in addressing the NCTM Standards in Grades K-4.

SE 052 164

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Volume 3 **Spring 1990**

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Prologue

This is the third and final monograph in the series. I want to express my great appreciation to Jim Boyd for the Bear Art, the Tech undergraduate and graduate students for their numerous and thought-provoking contributions, to Paula Buchanan and Terry Stevers, secretaries at Tech, and Teresa Thornton at Christiansburg Printing Company for their very fine work in helping me get these ready for printing. I'm also very indebted to the National Council of Teachers of Mathematics for their permission to reproduce articles which help translate and implement the Standards and to my wife, Ethel-Marie, for manning the bulk order and individual mail-outs.

Our work has just begun, and these monographs have just been a sprinkling of ideas. I encourage all who read these monographs to study the Standards, to join VCTM and NCTM, to participate in conferences, and to seek out other resources. Many resources are available from NCTM and are described inside the back cover.

Sincerely yours on behalf of
the VCTM Board and Officers,
Bob Underhill

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Three Ways Children Can Understand Math

Bob Underhill

Jerome Bruner made a very helpful contribution to our conceptual treatment of mathematics teaching and learning when he described *three ways* that we know. He called them *enactive* knowing, *iconic* knowing, and *symbolic* knowing.

Enactive Knowing

We know things *enactively* when they can be expressed through motor movement. If we can ride a bicycle, then we know *enactively* how to ride a bicycle. If we know how to water ski, then we know *enactively* how to water ski. This idea is useful in teaching and learning mathematics. If a child can clap three times when requested to do so, we can say that the child knows three enactively. If children can make circular motions in the air with their fingers, we can say they know circle enactively. If they can place two one-third pieces on top of a circular region, we can say they know two-thirds enactively. Sometimes we may feel that we are stretching the point a bit, but it is still a very useful concept. When children can do something *motorically which embodies the math concept*, we say they know the concept enactively. Doing something three times is an enactive representation of the *number* three. If we write "3," that is enactive knowledge of the *symbol or numeral*; it is *not* enactive knowledge of "threeness," the *number* three. A very young child can write "3" without knowing three just as the child can write "!" or "@" without having knowledge of the concepts for which these symbols stand as referents. We can say the same thing about "three;" it is possible to write this series of letters (knowing the letter series t-h-r-e-e) enactively without knowing the *concept* (threeness) enactively.

As teachers, we want to know how to help children know mathematics *concepts* enactively. Quite straightforwardly, we can see that learning activities which have major motoric components *which embody the mathematical concepts* help children know mathematics-concepts enactively. If we want children to know "2 + 3" enactively, we can have them make a set of two, make a set of three, and then point to each one as they count, "1, 2, 3, 4, 5." This activity has a major component which helps the children *know addition enactively*. If we want children to know "square" enactively, we can have them engage in tracing and drawing activities. And if we want them to know one-half enactively, we can have them engage in halving activities with crackers, fraction pieces, paper tearing, and so on. If we want them to know *fraction numerals* enactively, we can have them practice writing fractions. Please note, though, that this does *not* help them know fractions *conceptually*, only as names or numerals or symbols.

Iconic Knowing

If you know something *iconically*, you can create a mental picture. If you know the President of the United States iconically, you can picture his face in your mind. If you know your colleagues of the building where you teach, you can picture these in your mind, too. If you know how to ride a bicycle iconically, you can picture yourself or someone else riding a bicycle. Iconic knowledge is certainly different from motoric or enactive knowledge; just because you can picture riding a bicycle in your mind doesn't mean you can *do* it!

We want children to know mathematics *concepts* iconically. If children know the *concept of circle* iconically, they can imagine a circle or something which has the shape of a circle. If children know the *concept of three* iconically, they can visualize a set of three somethings; it might be three fish or three girls or three cars or it might even be three tallies or three circles. If children know the *concept of one-third* iconically, they can picture one-third of a circular region, or one-third of a cup of water, or one red marble and two blue marbles, or one-third of a candy-bar, and so on.

If children can visualize the word "himmel," they know the word himmel iconically. The word *represents the concept*. They don't know the *concept* iconically unless they know that this is the German word for cloud and can picture a cloud in their minds. Similarly, the symbols "12," "twelve," and "XII" can be known iconically, but, again, it is possible to know these *symbols* iconically without knowing the *concept* for which these symbols are referents, twelve-ness.

As teachers we want our students to *know math concepts iconically*. We can create activities which will *help them create mental pictures or images* to go with concepts. Thus, the children know ½ + ⅓ iconically if they can visualize ⅓ and ½ of a region and "see" that it is more than ½ and less than 1. To facilitate this iconic knowing, we can use instructional activities or learning experiences which help children create mental images or "picture in their minds" objects or actions which exemplify math concepts.

To help children have iconic *conceptual* knowledge of 2 + 3, we can have them engage in a wide variety of *visual experiences* which they can later visualize in their minds. Among these are: (1) using counters to find sums, (2) using pictures of animals or other objects to find sums, (3) watching TV, movies, or filmstrips *of objects* to find sums, (4) watching peers or teachers *use objects* to find sums, (5) looking at pictures in the book *which picture objects to demonstrate sums*, (6) drawing pictures or diagrams to demonstrate sums, and so on.

Please note once again that when children see only symbols without objects or pictures they learn only the *symbols* iconically. Such experiences do not help children develop iconic *conceptual* knowledge.

Symbolic Knowledge

We have two major symbol systems: language and mathematics. It can be stated unequivocally that children's symbolic knowledge should *always* be tied together with *conceptual enactive and iconic* knowledge. What is the point in knowing "ball" and being able to pronounce it (symbolic) if the learner doesn't know what a ball is (conceptual)? What is the point of knowing "circle" (symbolic) if the child can't imagine what a circle is (conceptual)? What is the point of know-

ing "5" or "five" if the child can't imagine an activity or a set with five-ness (conceptual)? What is the point in knowing "½ + ⅓" if the child can't imagine a half and a third of *something* (conceptual)?

Summary

We want learners to know mathematics concepts enactively, iconically, and symbolically. Some activities do two or even all three of them simultaneously. If the children talk about circles, trace circular objects and write "circle," they are learning circle all three ways. If fourth graders write 3/8 + 2/8 = 5/8 and manipulate fractional parts to demonstrate it is true or to decide the sum, they are learning sums of fractions enactively, iconically, and symbolically, all in one experience.

Our ultimate aim is to have children call upon their enactive and iconic knowledge when they encounter mathematical symbols. This represents one major standard against which we can measure understanding.

Now in the case that the least common denominator...
Professor Bear is a wonderfully creative mathematician...
Indeed... with every lecture he creates udder confusion!
JN BOYD

Perspectives on Teaching & Learning Math

Bob Underhill

The broad goal of these monographs is to help you teach mathematics. There is more than one perspective about how this can best be done. There are two major perspectives concerning learning in mathematics education today. One view proposes that mathematics exists in books and in the minds of some people. Within this perspective, the job of the teacher is to transfer this mathematical knowledge from his or her mind and from the book into the mind of the child. From this perspective, it is easy to see the dominance of behavioral objectives. The teacher knows how to find equivalent fractions; the book demonstrates how to find equivalent fractions, and the teacher views his or her job as showing the students how to find equivalent fractions.

Therefore, the teacher may write on the overhead, "At the end of the class, you will be able to name three equivalent fractions when given a fraction." The teacher may define equivalent fractions and demonstrate how to find equivalent fractions on the overhead or the chalkboard, and the students will practice so that they, too, can find equivalent fractions. In this approach, there are certain implicit assumptions that children arrive at the class with sufficiently similar knowledge that most of them can benefit and learn from the experiences planned by the teacher in such a way that they can all attain the objective.

I think it can be fairly stated that nearly all of us view understanding on a continuum. To express this in its most dramatic form, we might use the schematic in Figure 1.

FIGURE 1. Understanding

Now we would be hard-pressed to define what these absolute and complete categories are, but I think at some level we still have some sense about what the continuum means. And, when you teach, you expect (hope?) that the learner will edge a little bit to the right. Hopefully, over time the learner will move quite a bit to the right.

One of the great challenges of teaching is keeping *everyone* moving to the right. This means that learning experiences must be sufficiently rich to be in some manner within the grasp of every learner. So what is *rich*? One way to articulate its meaning is to imagine a set of learners and a set of activities in the classroom. The environment is *rich* if every learner can at some level make sense of what is going on in such a way that he or she can move to the right on our continuum.

We might draw the analogy two different ways. First, the teacher might be viewed as a junction in a pipeline. There are 25 lines running from the teacher, and the teacher has the responsibility to ensure that what is flowing through each one of those pipes is producing movement on our continuum. In classrooms there is frequently a one-way flow between the teacher and each pupil. This makes the teacher's task pretty awesome.

The second analogy might view the teacher as the conductor of an orchestra. The teacher guides the progress of each player, but there is a multi-faceted exchange of information. The conductor directs based on the musical score and what she knows of each player and each section; each player responds to his musical score, the collective sound from his section; and the direction of the conductor. Directing a symphony is *also* an awesome task! (You thought I was going to make the second one sound *easy*, didn't you?!)

We're all human. We know that when we are confronted with awesome tasks that we *will* find a way to manage. And we do. When we manage, we make compromises which achieve a satisfactory balance among our skills, our resources, and our partners.

FIGURE 2. The most common way math teaching is being done

In conventional teaching (see Figure 2), the teacher seeks to maximize her teaching effectiveness by carefully selecting and using her personal and curricular resources to deliver instruction to her pupils. For example, our teacher will use the textbook, chalkboard, and fraction-embedded objects to teach about equivalent fractions in as clear and concise an exposition as possible to produce learning. The compromises are many because of the human limitations in this setting:

1. The teacher can do only one thing at a time.
2. There is limited knowledge about where each learner is on the continuum.
3. Whatever is going on can only be optimally appropriate for *some* of the learners.

One of the things we want to do is look for ways to enhance this conventional model. It certainly does work for many people. Probably you and I had most of our mathematics instruction in this model. I don't know about you, but I got good marks in math. Even so, there were many times when I resorted to memorizing things because it was the only way I could do what the teacher wanted on the test. I didn't understand, but I got by.

There is an implicit assumption in this model which we need to examine: *We can only learn from experts*. This assumption is acted out in traditional classrooms all over the world. We, as teachers, often behave as though learning is brought about by us (we are the resident experts) and by the contents of a book (which was written by a distant set of experts).

On many occasions when I have been a student, I didn't rely on experts.

1. my resident expert wasn't always available to me;

2. my distant experts weren't always comprehensible to me;
3. even when my resident expert *was* available, I and the expert couldn't always get on the same wave length.

My teacher just didn't understand me (sniff)!

I always turned to a friend when I found myself in this situation. And, guess what, I nearly always turned to a friend in the *same class*! We were *both novices*. Studying with a friend was nearly always something which helped. As we tried to understand we could often be helpful to each other. Why did this work? The main reason is because we both understood bits and pieces but they were often *different* bits and pieces. Thus, when we studied together, it was like each of us had some pieces of the larger puzzle. I could fill in some of his missing pieces, and he could fill in some of my missing pieces!

So, the question I leave you with is this: If we all learn from experts *and* novices, why don't we capitalize on *both* of these in the classroom?

Children's Views of Mathematics

After completing a master's level course at Tech, teachers were asked to describe "a rich conception of mathematics as it might be known to a child." Here are some responses.

Brett Hale - the child listens to the story that utilizes some mathematical concept. The child then conceptualizes what factors in the story are to be acted on; the child makes the *decision as to whether something will increase or decrease in number*, or will be sectioned up to make smaller groups or sets. In order to represent the change that is taking place as a result of the story, the child will *utilize manipulatives to give visual, tactile, and kinesthetic properties to the* concepts that are being worked with in the child's mind. If the child utilizes the manipulatives, the processes that are at work with the *manipulatives* are transferred to *pictures on paper* that are compared with number algorithms. Finally, the child is able to *transfer the functions* that are at work with the interactions of the objects or concepts in question from the story to purely abstract numerals. The child then reviews the process in order to make sure the results are arrived at to personal satisfaction.

Liz Hiller - children who have a rich conception of mathematics *believe that math is useful in their everyday* lives. When they describe occasions in which they use math, instead of exclusively mentioning activities found in a traditional classroom such as doing worksheets, completing the problems in the book, they would mention activities such as *finding out how much more* money they need to buy a skateboard, which piece of candy was the best bargain, what their batting average is, and how many more days of school are left until summer vacation. To these students math *is purposeful, not isolated*, skills found in a math book.

What Do Teachers Do?

Here are some brief ruminations on what teachers can do to help young children develop richer conceptions:

Bill Turner - a teacher can do much to influence a child's conception of mathematics learning. The math learning experience must be one that the child can *associate with positive feelings and memories*. The teacher is largely responsible for this. First of all, the teacher must provide the *why* of learning something before the how. If a child can see a reason for being taught a new concept, then he is more likely to *want to learn it*. Children need to be led to believe that math is something that is *part of our everyday lives*, not some tedious task that must be performed in school. Teachers need to make math *as enjoyable* as possible, creating *a low-risk environment* for learning. Allowing students to *work together* to arrive at solutions is also helpful in forming positive feelings about math learning. Teachers also need to be *readily available to assist* children in the classroom. They need to know that *any serious question is legitimate and important*. Something that teachers should not do is assign too many practice problems after a concept is learned. She should not send children home with assignments involving a skill with which they *are not familiar*. Teachers should design learning so that work well done *is the reinforcer rather than the grade*.

Bob Underhill - facilitates; challenges; uses wait-time; explores; helps; questions; problem-poses; directs to resources; enables; uses formative assessment (or diag.); cognitivist approach; constructivist orientation; fosters independence; seeks to create cognitive disequilibrium; reflects about teaching and learning; varies teaching strategies; avoids "schoolish" questions; uses cooperative *and* competitive grouping strategies; expects *all* learners to grow mathematically; empowers learners; enhances self-esteem and self-efficacy as "mathematical beings"; pushes for relational understanding; developing intrinsic motivation; uses real life problems; stimulates peer interaction; paired problem solving; discussions of strategies; focuses on *processes* as well as answers; helps students recognize and deal with misconceptions; "Sharing" opportunities; models good problem solving; uses manipulatives; attends to affective as well as cognitive aspects; discovery; pushes for making connections; emphasizes patterns; ties memory with understanding; relates to child's experiences and interest; formulates "theories" or "models" or learners' knowledge; is sensitive to development (psyc); uses a variety of manipulatives; encourages and accepts diversity.

Math Teaching from a Cognitivist Perspective

Math teaching has been dominated by behaviorist approaches. Researchers have found that cognitivist approaches are more successful at developing higher order thinking skills. Here are a couple lesson plans to communicate the flavor or tenor of such lessons.

Lesson I - *Natalie Newman*

Objective: The objective of this lesson is to help children develop procedures for addition of same denominator fractions.

Materials: Cuisenaire Rods and overhead projector.

Methods:

1. Begin lesson with discussion of running. Ask if anyone enjoys running. How far do they run? How often? When? Where?
2. Introduce Math story problem: Tom ran 1 2/6 miles on Monday. On Friday he ran 2 5/6 miles. How many miles did he run in all?
3. Allowing students to work in pairs, hand them each a set of Cuisenaire Rods and tell them to find a way to solve the problem, using the rods if they so choose. Allow students a time allotment of twenty minutes to find a solution. Have them record their processes.
4. After twenty minutes, bring the class back together as a group and share everyone's solutions. Discuss each process they come up with, noting the different approaches taken. Reinforce the idea that no approach is the only correct method. (Use overhead so that whole class can observe.)
5. Reinforce concept with more math story problems, and allow students to work on them.
6. Close lesson by reviewing the day's concepts and skills learned.

Lesson II - *Bill Turner*

Introduction to Division*

1. Engage the class in a discussion about sharing things, especially when there isn't enough of something for everyone to have more than one. Direct the discussion to several items distributed over a certain number of children. Perhaps a card game could be mentioned where all cards are distributed. Would each person get the same amount? How do we know?
2. Plan a Valentine's Day party for the class. Decide how many packages of various items (such as paper plates, candy, etc.) will be needed for the class of 24.
3. The teacher will have one package each of party items displayed at the front of the room. Students will be able to see how many of each item comes in a package. Students will work in groups of four. They are to decide what they want to have at the party and how many packages of each will be needed to supply the class. They will be free to move about the room using available supplies such as paper and scissors. When everyone is finished, one person from each group will share his plan with the rest of the class.
4. Remind students of past activities and how they were used to demonstrate subtraction and multiplication, which are related to what they will do here. Lead them to think about this while planning their party.
5. Give an example. Perhaps point out a package of six candy hearts. How many packages would be needed for each person to have one? Don't give the answer, but encourage them to figure it out amongst themselves.
6. Go around the room and observe activity. Give guidance, support and encouragement. Ask leading questions and cause them to draw on past knowledge and experiences. Help them think for themselves.
7. Have groups share their ideas. Look for any commonalities among groups and tie them together. Promote a discussion.
8. Most likely, not everyone will understand all that took place. Now is the time to clarify. Hopefully, some children used repeated subtraction to arrive at their conclusions. A child who clearly understood could explain to the class. The teacher should tie up loose ends using pictures and manipulatives. Prepare to transfer to symbolic representation. Ask the class to think of other situations where they could use what they have learned. These could be shared to begin the next session.

*Some children may approach this project through multiplication. This is fine as it will emerge later in the lesson that division and multiplication can often be used to answer a question depending on whether it is framed as $6x? = 24$ or $24 \div 6 = ?$.

Natalie and Bill were graduate students at Tech in the spring of 1989 when they developed these lesson plans.

STANDARD 10: MEASUREMENT

In grades K–4, the mathematics curriculum should include measurement so that students can—

- ***understand the attributes of length, capacity, weight, area, volume, time, temperature, and angle;***
- ***develop the process of measuring and concepts related to units of measurement;***
- ***make and use estimates of measurement;***
- ***make and use measurements in problem and everyday situations.***

Focus

Measurement is of central importance to the curriculum because of its power to help children see that mathematics is useful in everyday life and to help them develop many mathematical concepts and skills. Measuring is a natural context in which to introduce the need for learning about fractions and decimals, and it encourages children to be actively involved in solving and discussing problems.

Instruction at the K–4 level emphasizes the importance of establishing a firm foundation in the basic underlying concepts and skills of measurement. Children need to understand the attribute to be measured as well as what it means to measure. Before they are capable of such understanding, they must first experience a variety of activities that focus on comparing objects directly, covering them with various units, and counting the units. Premature use of instruments or formulas leaves children without the understanding necessary for solving measurement problems.

Estimation should be emphasized because it helps children understand the attributes and the process of measuring as well as gain an awareness of the sizes of units. Everyday situations in which only an estimate is required should be included. Since measurements are not exact, children should realize that it is often appropriate, for example, to report a measurement as between eight and nine centimeters or about three hours.

As measurement concepts and skills are introduced, they should be integrated throughout mathematics and other curriculum areas. Not only will this enhance other topics but it will also give children opportunities to develop and retain measurement concepts and skills.

STANDARD 10:

- **use problem-solving approaches to investigate and understand mathematical content;**
- **formulate problems from situations within and outside mathematics;**
- **develop and apply a variety of strategies to solve problems, with emphasis on multistep and nonroutine problems;**
- **verify and interpret results with respect to the original problem situation;**
- **generalize solutions and strategies to new problem situations;**
- **acquire confidence in using mathematics meaningfully.**

Activities for the Measurement Standard

1. Line up in order from shortest to tallest for picture taking. (Elyse Williams)

2. Bring in various boxes, containers that children encounter daily: Shoe boxes, cereal boxes, soup cans, etc. Have children measure and explore the volume, height, weight, etc. of each item. (Laura West)

3. Keeping in mind that children love boxes, have an activity with a fairly large box and have the children discuss ways and with various units the box could be measured. Encourage them not only to think of one side but also perimeter and volume (and whatever they think of). Then extend this by asking what could be measured by using the box as a unit of measurement. (Brenda Sloss)

4. Have the students measure the lengths of objects around the room using different items as the standard unit (i.e. crayon, pencil, their shoe, etc.). (Marnea Prestel)

5. I would use the activity called "A Heavy Matter." In this activity students compare objects like a nickel and a penny and estimate which one is heavier by placing each item in one hand. Then they find the actual weight by using a balance scale and gram weights. (Jennifer Hoelk)

6. I need to decide which room to have our party in. I would like to use the larger room. Try to find out which room is larger. Be prepared to tell me how you did it. (Deena Loan)

7. If you need to find out how long a table is and you didn't want to use a yard stick or measuring tape, you could help the children find the "length" of the table by using an everyday object such as a rope or blocks. You could also find the area of the table using blocks or even sheets of paper. This allows the students to visualize what length and area are. This also builds readiness skills. (Melissa Baker)

8. The teacher could set up learning centers around the room. At each center the students with practice estimation and measurement. For example, one learning center could have several rectangular shaped boxes. AS units of measurement, 1 in. cubes could be used. Questions such as how many cubes (units) is the object in *your* opinion? Now, find out how many it actually is. Was your estimation close? (Ame Harrell)

These ideas are from Tech students in the K-8 certification program.

Measurement Skills of Young Children

Liz Hiller and Marilyn Weih

To determine how children conserve when working with measurement we interviewed two first graders, Frederick and Frank, about how to measure length. We told the children that we wanted to move the picnic table into the house for the winter; would the table fit through the door? They didn't think it would fit, but when asked how they could tell for sure, they said it could be measured. They found a large ball of string and proceeded to measure the length of the table. Then they used the same string to measure the height of the doorway. However, neither held the mark on the string when they pulled away from the table or doorway. Then they thought about the legs which form a cross and so tried to measure these, but on the diagonal, and became confused. When we suggested that they measure the width of the table, they did, but then started to measure around it, and again became confused.

When Frederick and Frank were asked for another way to measure the table, they suggested using a brick or stick. Frederick brought a stick to the table and concluded that it was too small. Then Frank used a baseball bat. However, when the baseball bat proved shorter than the table, they said they could not use it. We suggested counting how many bat lengths were needed to measure the table. They did this but did not measure accurately and did not use fractions, only whole numbers. This was very confusing to them.

Next, we asked the children to see if a filing cabinet would fit in a certain space. The children found a collection of different toys which they stacked up next to the filing cabinet to see how tall it was. They then stacked the same toys in the same order in the other space and decided that the cabinet would fit. The children always selected to measure the height rather than the width which was the real measure in question.

When the children were asked to measure the width of the filing cabinet with only one toy, they said they could not because the toy was too small. When it was suggested that they use counting and were shown an example, they immediately moved a puzzle box along, again not noting the end nor the beginning. They correctly concluded the filing cabinet would fit.

Using two roller skates placed side by side, the children correctly identified that the skates were the same length. But when one skate was moved forward, both children now thought that the skates were of different length.

Next, Frederick and Frank were able to tell that two paper clip chains placed side by side was the same length. When one chain was moved ahead an inch, both children said that it was now longer or bigger than the other chain. However, when the two halves of the chain were laid end to end and next to another whole chain, the children did agree that both the whole chain and the divided chain were the same size.

Finally, we told the children a story about a turtle and a rabbit both going to town. The turtle went straight to town but the rabbit took a round-about, zig-zagging path to town. The rabbit and the turtle both started and ended at the same place. This story was accompanied by a drawing. Both children said that the rabbit took the longer route, which was correct. This was further substantiated by the children using a paper clip chain to measure the routes of each animal. However, when the children were asked about the distance from the animal's homes to the town, they were slightly confused, but eventually agreed that the distance was the same.

A week and a half later we again tested the boys about measuring. This time we questioned them separately. The picnic table was brought closer to the door in such a position as to demonstrate how the table would be brought into the house. Frank began by measuring the length with the string and then switched to the width. He was not careful about marking the beginning and end. He laid the string out on the table and counted as if the string were marked in one inch intervals. When he measured the width of the door, he did the same. When it was suggested to Frank that he use a trowel to measure the table he remembered to count the lengths of the trowel needed to measure the table. He was not very accurate in lining up the trowel but still found that the table would fit.

Frederick still measured the length of the table and the height of the door as he had on the previous occasion. He laid down the string after each measure without marking it in any way.

Next we asked both boys if a large spatula would fit in a drawer. Frank used a tape measure to measure the open drawer along the outside, not noticing that the inside was smaller. While he did use the beginning of the tape measure as a starting point, he did not align it with one end of the spatula. From this, Frank decided that the spatula would fit in the drawer.

Frederick pulled out the drawer realizing he would have to measure the inside dimensions of the drawer. He measured both the drawer and the utensil with a string and the tape measure. Not measuring very accurately, he decided that it would fit. Then when he tried it, it did not fit. So he measured it again and said it would fit but when he tried it, it still did not fit. This was confusing to him.

Lastly, we asked each boy about the paper clip chains again. Two chains were laid side by side and it was established that they were the same length. Then one chain was divided in two and separated and each boy was asked, "If the two pieces of chain were added together, would they be the same as the other chain?" Both boys agreed the chains would be of equal length.

Conclusions

A confusing area for the boys seemed to be the measuring of consistent sides. We were surprised that the boys measured the length of the picnic table and the height of the door, as if the table would be turned on its end and carried into the house. Obviously, they did not consider or could not visualize how the table would be carried into the house. In addition, they were concerned with the height of the filing cabinet instead of the width. They seemed to relate the concept of length mainly to height.

It was interesting that the boys could line up a variety of objects, but could not accurately move one small object end to measure length. This seemed to show that they could not visualize the space taken up by an object unless the object was actually there.

Both children appear to be at a transitional level of conserving length as demonstrated by the activities with the roller skates and the paper clip chains. They did have a basic grasp of additivity of length.

A problem with understanding may partially stem from vocabulary. We used various words in our comparisons, including big, long, and length. The children understood big, but did not clearly understand long and length. It could be that they have to understand the *concept* of length before they know the *words*.

It appeared that both Frederick and Frank have some aspects of the concept of measurement. We assume that this has come from observations of their parents and teachers. They understand that sizes of objects are compared and lengths can be added to form equal sizes. They recognize that a tool is used (a tape measure) to make these measurements. They do not seem comfortable with the concept of an iterative (repeated) unit. We believe one of the last basic concepts to be understood is that of accurate and careful measurement.

Primary level teachers would do well to focus their early instruction on non-numeric, non-standard measurement. This is a vital building block for complete understanding of measurement. Right from the beginning, it is important to expose children to many instances of measuring. Working together with a partner or in teams will help them discover alternatives to their own strategies.

Liz and Marilyn wrote this article as a part of their graduate course experience at Virginia Tech.

Professor Bear Can Never Stay Ahead of Kangaroos

So x must be approx i...
x + 1/x
mately 7.091735216
Kangaroos Never leave home without a pouch calculator
You just can't keep those kangaroos down
JNBOLD

They Always Have a Jump on Him.

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Metric Song (Tune: Locomotion)

Roberta Sherman

All arond the world people have to measure.
Come on children, try the metric system.
Distance, speed, weight, length and volume, even weather
Come on children, use the metric system.
Our country is one of a few we do know
That uses different measurement, but even so
Come on, come on use the metric system with me.

You got to learn the terms now.
Come on, let's see, you just repeat after me:

Oh, Oh (Refrain)

Millimeter, centimeter, decimeter
Meter, kilometer,
Milligram, centigram, decigram,
Gram, kilogram.

For measuring the weather everyone should know,
If it is zero Celsius, it just might snow.
So come on, come on use the metric system with me.

You got to learn the terms now.
Come on, let's see, you just repeat after me:

Oh, Oh (Refrain)

If you need to measure liquids for the cake you're baking,
Come on children use the metric system.
Three metric units will help produce the cake you're making.
Come on children, use the metric system.

Milliliter, liter, kiloliter are thee.
Units that are easy, just try them and see.
So come on, come on use the metric system with me.

It seems you learned the terms now.
Come on, you see how easy learning it can be:

Oh, Oh (Refrain)

Metric is so simple it is based on ten.
Come on children use the metric system.
You'll find that you will use it again and again.
Come on children use the metric system.
The more you practice you will see it's easy to learn.
Remember, if is 30 Celcius our skin will burn.
So come on, come on use the metric system with me.

Come on, Come on its easy just you try it and see!

Roberta teachers at Great Falls Elementary School.

For Years Professor Bear's Colleagues Have Felt That Some of his Classes Were for the Birds

Although Young Owlbert Was Dozing, the New Standards Will Produce Wise Young Owls! And the Owly Bird will always get his Worm.

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FOR YOUR CLASSROOM: PRIMARY
TAKE LOTS OF TIME TO TEACH TIME

Lois F. Brown
Thomas Jefferson School
Wausau, Wisconsin

To be most effective time should be taught throughout the school year, not solely in a 2 or 3 week block of time. However, math guides are often lacking in sufficient activities to help teach time to the young student. Following are some suggestions that the primary teacher might want to use sometime between September and June, depending upon the needs of his/her particular class.

Develop an interest in clocks:
- Try to visit a clock store.
- Bring in different kinds of clocks to show and display for a day.

Counting by 1s and 5s:
- Play BUZZ. Have the children count by ones, with each child supplying one number. Next tell the students that each time the number ends in 5 or 0 the person says, "Buzz" instead of the number. Those who say the number instead of "Buzz" are out. Once they reach 60 start over at 1 again. Those who are left at the end of the second round are the winners.
- Play "Hide and Seek" in the classroom. "It" closes his/her eyes and counts by fives from 5 to 60 while another child hides something in the room. The rest of the class counts by 5 to 60 twice, as "It" searches for the hidden object.
- Have students practice "counting on" by 1s after multiples of 5. First, lead them in counting by 5 by opening and closing 1 hand quickly - thus "flashing" fives. Each time your hand opens the student should count another 5 fingers:
  5, 10, 15, 20
  After several such flashes, hold up 1 finger at a time to initiate counting on by 1s. This might be one counting sequence:
  5, 10, 15, 20, 21, 22, 23
  Vary the number of fives between zero and eleven and the number of ones between 1 and 4 as you repeat the activity.
- Construct 2 number lines labeled 0 - 12 and 0 - 60. Show the students how the numberline numerals correspond with those on a clockface on which the numerals 1 - 12 and 5 - 60 are listed.
- Practice the hour-number and minute-number pairs to help the students tell time without the aid of the minute rims on clocks. Write an hour number on the board. Then ask for a minute number and write it next to the hour number. Repeat the procedure as often as you think necessary with other hour and minute numbers. The order should always be hour number shown; minute number asked for.

Timing activities:
- With a stopwatch or clock with a second hand students time activities such as:
  Counting from one to one hundred
  Doing 10 jumping jacks
  Singing a song
  Cleaning up room
  Lining up
  Washing hands
  Saying a nursery rhyme
  Putting on boots
- Have students discover what they can do in 5 minutes. Each student chooses activity to do. The children should note the time on the clock when teacher says "GO!" to start activity and when teacher says "STOP!" to stop activity. Volunteers tell what they did in 5 minutes. Students discuss if the 5 minute period seemed long or short and why.
- Teacher sets timer or alarm clock to ring after a certain designated period of time. Children should note starting and stopping time. Teacher says the length of time the activity will take place..."We will sing for ten minutes."

Telling time on the hour, half hour, or time being studied:
- Show a clock face. Give each of 12 children a numeral card. Help them place the numerals on the floor in a circular pattern that looks like the face of a clock. Make a knot in a jump rope closer to one handle than the other. Place the knot at the center of the circle and the long part of the rope with the handle pointing to 12. Ask the children to show various times on the hour by moving the short "hand." Either show times on the demonstration clock or call them out.
- Each child has a paper clock. They are asked to set their clocks on any hour. As each name is called the child says, "My clock says _____ o'clock," and tells some activity that (s)he usually does at that hour.
- Play Match the Hour with the clocks. Divide children in teams of 3. Say, "I am thinking of a time between 4 o'clock and 6 o'clock." Have each team member try to guess what time is on the clocks. Hold up an index card showing the time being guessed, such as 5:00. Each child who has a matching hour scores one point. The team with the most points wins.
- Divide the children into "families" and let them make up situations which require the use of a clock. Have them make a statement or ask a question and show the time on the clock.

Father: I'll be home for supper by 6:00
Mother: Sue, you have to go to the dentist at 3:00 this afternoon.
Sue: I'm going to have breakfast at 7:00 today.
Bob: My lunch hour is 11:30

- Play Story Time: Examples:
  - We start to play at 9:00 and stopped an hour later. What time was that? Show it on your clocks.
  - It takes one hour to drive to Stevens Point. If we leave school by 8:00 what time should we get there?
  - At 4:00 Jenny asks if she can go to Kelly's to play. Her mother says she may if she will be home in an hour. When should Jenny be home?

- Prepare clock faces showing times to the hour and half hour. Divide the students into 2 teams. Place the cards face down on a tray and let a member of the first team draw a card, hold it up, and tell the time. Alternate teams and keep score.

- Appoint TIME CALLERS. They are students who ring a bell and call out time on the hour, half hour, or time being studied..."It's 2:30 in room 11!"

- Make paper plate clocks to be used for designated times, such as recess, dismissal, arrival time of specialized teachers, etc.

- SET THE CLOCK GAME. A clock with moveable hands is needed. Cut 15 3"x3" pieces of tagboard. On the front of each card write a time. On the back of each card draw a clock face showing the time. Game could be played by 1 or 2 children. Cards are turned so player sees just the time. He looks at the time written on the first card, and sets the clock to show that time. Then he turns the card over to check his work. His clock should look exactly like the one in the picture.

- CLOCK BINGO. Teacher hands out dittoed sheets as the bingo cards. The cards should show different times in digit form and on clock faces. All times should be in five minute intervals. Read off different times and have the children mark their cards. Any straight line wins.

- MIDNIGHT. 2 to 12 may play.
  Materials: Two packs of cards. On each card is a clock face showing a certain time and below it is written the time. No two cards are alike. The second pack is like the first except that the time is not written on each card. Markers are needed for each player.
  Cards without time written on them are dealt face up. Each player has the same number of cards. The caller who has the cards with time shown and written holds his cards and reads them one at a time. If he has the appropriate clock, the player places a marker on the clock showing the time called. The first player to cover all his cards calls "Midnight." The caller then checks the player to see if he is correct by having him read back the time on each clock. As the player reads, the caller must find the corresponding card among the cards called during the game.

- CLOCK CONCENTRATION.
  Materials: six 3"x5" cards with clock faces that show 3:00, 7:00, 12:00, 4:30, 6:30, 10:30 (or any times being studied). Six 3"x5" cards on which those times are written. Two children at a time play the game. They mix the cards and place them in three rows, facedown. The first player turns up two cards. If they match, the player keeps them. If they do not match, the cards are returned to their original facedown positions. The next player takes a turn and does the same. Any player who makes a match gets another turn. The player with the most cards at the end of the game is the winner.

- Collect a number of pictures showing ordinary daily activities, for example, eating breakfast, catching the bus, playing outdoors. As you show a picture, call on a student to show on the demonstration clock the time the activity might occur. If the child shows time on the hour, ask someone to show how the hands would be placed a half hour later.

- COUNTING THE MINUTES.
  Move the minute hand on a demonstration clock and ask individuals to help with the counting.
  "The minute hand goes
  1, 2, 3, 4, 5, STOP!
  Five minutes after twelve o'clock.

  The minute hand goes 6, 7, 8, 9, 10, STOP!
  Ten minutes after twelve o'clock."
  Continue until 30 minutes after twelve is reached.

- TELLING TIME BOOKLETS. Fold 12"x18" paper into fourths, cut on lines, making a four-page booklet. Top sheet is cover. Other sheets each tell about the times that are important to the child in his day. He writes a sentence on each sheet such as "I go to school at 8:00" and draws a clock face above the sentence showing 8:00.

- Game Rhyme:
  WHAT TIME DO WE HAVE?
  "When the short hand is on 3 (place short hand on 3)
  and the long hand is on 12 (place long hand on 12)
  Think about it!
  What time do we have?"
  (Children say in unison: "3 o'clock")
  Rhyme is repeated with other hours, letting individual children manipulate the hands of the clock.

Measurement Experiences in Grades K-4

Liz Hiller

In grades K-4, the mathematics curriculum includes measurement so that students can: understand the attributes of length, capacity, weight, area, volume, time, temperature, and angle; develop the process of measuring and concepts related to units of measurement; make and use estimates of measurement; and make and use measurements in problem and everyday situations.

Many teachers erroneously believe that students can measure with standard tools from an early age and with maturity comes the ability to measure in smaller and smaller increments. Instead, students need to explore measurements concepts before using the standard tools. They need opportunities for comparing the sizes of different objects, by manipulating these objects and developing nonstandard tools of measurements. In doing so students will then develop the vocabulary associated with measurement and they will develop intuitive concepts. Words such as longer, heavier, taller, larger, perimeter, side, length, depth, weight, mass, and area will then have greater meaning because they can then be applied to the object and the problem at hand.

Measuring becomes a useful activity if the experiences are real and purposeful. "Schoolish" questions, such as "How long is your pencil?" creates a task that leaves the students wondering why you want to know. Instead, you can involve your students in activities such as determining whether the art paper is too big for their desks and whether they will have to move it to a table, whether the books are tall enough to be stored on a shelf, or if empty space in the classroom is large enough for one more bookshelf. Real, everyday measuring activities help create the sense of applicability.

You can emphasize estimation in measurement by allowing the students to give answers that are reasonable. During activities in which accuracy is unnecessary, students should be allowed to give lengths in whole numbers, instead of fractions. Everyday situations which encourage estimation should be included in measurement. Problems involving time and temperature can easily be used.

In allowing students to begin measurement by exploring objects and their common attributes and by using real world situations you will be allowing your students to take control of their learning and to develop understanding at their own rate. With these positive attitudes developing at a young age, measurement will become more than using the rule to measure the picture of the pencil in the text. Students will make connections to their everyday lives and make comparisons and measurements throughout their day.

Liz teaches in Arlington, Virginia.

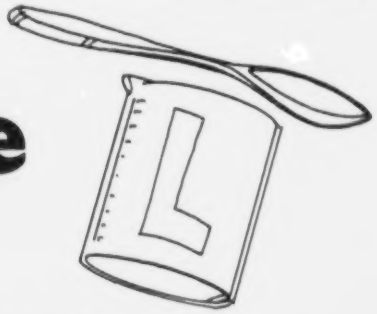
Our gallant knight, Sir Lancelot
Rode about on his horse a lot.
He searched around
'Til he had found
Teachers brave enough to chance a lot.

Sir Lancelot Bear Seeketh the Standard Bear-ers

IDEAS

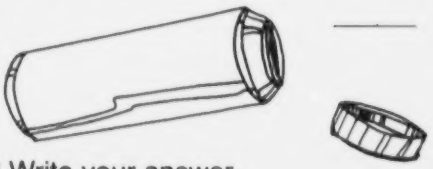
Name _____

Fill It Up, Please



Try these experiments!

1. Fill a glass with spoonfuls of something. How many spoonfuls did it take? _____
2. Fill a liter container with glassfuls of something. How many did it take? _____
3. Fill a bowl with liters of something. How many did it take? _____



Make a guess!

Do you think that each of these holds *more* or *less* than a liter? Write your answer.

a glass _____ a sandbox _____

a fish bowl _____ a carton of milk _____

a sink _____ a can of soup _____

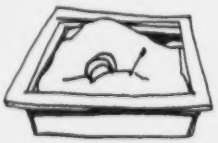
a Thermos bottle _____ a bucket _____

Test your answers to see if you are right.

How many does it take to make a liter?

What we measured with	My guess	Our measure
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____

Bonus!



What we measured	My guess	Our measure
_____	_____	_____

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STANDARD 11: STATISTICS AND PROBABILITY

In grades K–4, the mathematics curriculum should include experiences with data analysis and probability so that students can—

- ***collect, organize, and describe data;***
- ***construct, read, and interpret displays of data;***
- ***formulate and solve problems that involve collecting and analyzing data;***
- ***explore concepts of chance.***

Focus

Collecting, organizing, describing, displaying, and interpreting data, as well as making decisions and predictions on the basis of that information, are skills that are increasingly important in a society based on technology and communication. These processes are particularly appropriate for young children because they can be used to solve problems that often are inherently interesting, represent significant applications of mathematics to practical questions, and offer rich opportunities for mathematical inquiry. The study of statistics and probability highlights the importance of questioning, conjecturing, and searching for relationships when formulating and solving real-world problems.

A spirit of investigation and exploration should permeate statistics instruction. Children's questions about the physical world can often be answered by collecting and analyzing data. After generating questions, they decide what information is appropriate and how it can be collected, displayed, and interpreted to answer their questions. The analysis and evaluation that occur as children attempt to draw conclusions about the original problem often lead to new conjectures and productive investigations. This entire process broadens children's views of mathematics and its usefulness.

Statistics and probability are important links to other content areas, such as social studies and science. They also can reinforce communication skills as children discuss and write about their activities and their conclusions. Within mathematics, these topics regularly involve the uses of number, measurement, estimation, and problem solving.

STANDARD 11:

- **use problem-solving approaches to investigate and understand mathematical content;**
- **formulate problems from situations within and outside mathematics;**
- **develop and apply a variety of strategies to solve problems, with emphasis on multistep and nonroutine problems;**
- **verify and interpret results with respect to the original problem situation;**
- **generalize solutions and strategies to new problem situations;**
- **acquire confidence in using mathematics meaningfully.**

Activities for the Statistics and Probability Standard

1. Have the students in groups of two. Give each pair of students a bag with two white balls in it and one black ball. (Marbles can be used instead of balls). Have the students take turns picking one ball from the bag and putting it back in the bag. The students are to make a chart of their draws. After a sufficient amount of draws have the students look at their charts. Do they notice any patterns? Which color ball was drawn the most? What were the chances of picking a black ball as opposed to picking a white ball? (Christine Whitley)

2. Go around the classroom interviewing each other. Find out what veggies each person likes and hates. After completing that, construct a bar graph to present your results/findings. Ex. How many people like and dislike each vegetable. (Pauline Payette)

3. Hand out snack size packages of M&M's. Have the kids get into groups of 4. Make a chart recording the number of each color in each package. Then ask them questions about the chances of pulling out a green M&M when first reaching into the bag. Have them experiment to design ways to answer the questions. (Elyse Williams)

4. Around election time a good activity would be to have the students poll an area for who they would support. With this find the total number of people polled and what fraction supported each candidate. Then pose the question "Suppose 93,000 people vote for one of these candidates. How many will vote for each one? (Brenda Sloss)

5. Have children work in groups. Have each group decide on a survey of the class that they would like to conduct (i.e. favorite kind of pudding, etc.) Have each group decide on a way to present the information to class. Provide them with materials to make displays. (Marnea Prestel)

6. Have class make a chart showing how many brothers and sisters they have. (Ask other questions too like favorite food, tv show, etc.). Then ask questions like who has the most brothers? Sisters? Looking at the totals, does our class have more brothers or sisters? Do this questioning with each category. (Jennifer Hoelk)

7. Ask students to take a poll of people in their class and at home to see what kinds of pop different people like. They could make a chart to show the results. (Kristin Givens)

8. The people in the cafeteria need to know if they should order more chocolate, whole, or skim milk. Have students devise a way to tell. One way is to collect all the milk cartons one day and construct a "graph" using the cartons on a table. (Jeanie Mohwa)

9. If you are in a dark room and you reach into a cooler of sodas in which there are 10 Pepsi, 6 Mountain Dew, 13 Coke, 4 Gingerale and 8 Root Beer. What is the chance you will pick out a Mountain Dew? Pepsi? Coke? Gingerale? Root Beer? Have them work in pairs and make many draws of marbles with colors to reflect those numbers. (Lori M. Castello)

10. We want to have our own cola taste test. The choices are Pepsi, Cole, and other. Let's make a graph to decide which cola we should get for our class party. (Donna Davis)

These ideas are from Tech students in the K-8 certification program.

M&M Statistics and Probability

by Paul Giganti

There's more in a bag of M&M's than just candy. Here are some statistics and probability experiments that students of all ages can perform with a bag of M&M's. These activities work best and cost less when done cooperatively in groups of four.

1. Give each group of students one small bag of M&M's and one loose M&M to use as a sample. Caution them not to open their bag yet. Explain to them that they are going to make some mathematical guesses about what is inside their bag of M&M's without dumping the bag out and looking at the M&M's. They will use estimation and sampling. Tell them that this is the kind of guessing that poll takers must make before an election.

2. Their first task is to estimate how many M&M's are in the bag without opening it. The sample M&M can be used as a tool if they wish. Have them report their estimate and how they arrived at their answer. Average all of the groups estimates to get a class estimate.

3. Now have each group open their bag of M&M's and put it into a small brown bag. No one gets to look into the bag, but each will now take a turn sticking their hand in and drawing out just one M&M. The group will do this at least thirty times and can use a copy of the M&M Graph page to record each trial. Each time the M&M must then be placed back into the brown bag. Now have each group make an estimate of what proportion there is of each color M&M by looking carefully at their data.

4. Finally students get to dump out their M&M's onto the table and count them. They should also color in another M&M Graph with the actual number of each color M&M. Also take a class average of the actual number of M&M's in a bag. Discuss the differences between the estimates of color and number and the actual counts. Were all the bags of M&M's the same? How is this activity like what pollsters have to do to predict elections.

Reprinted from The CMC ComMuniCator, California Mathematics Council.

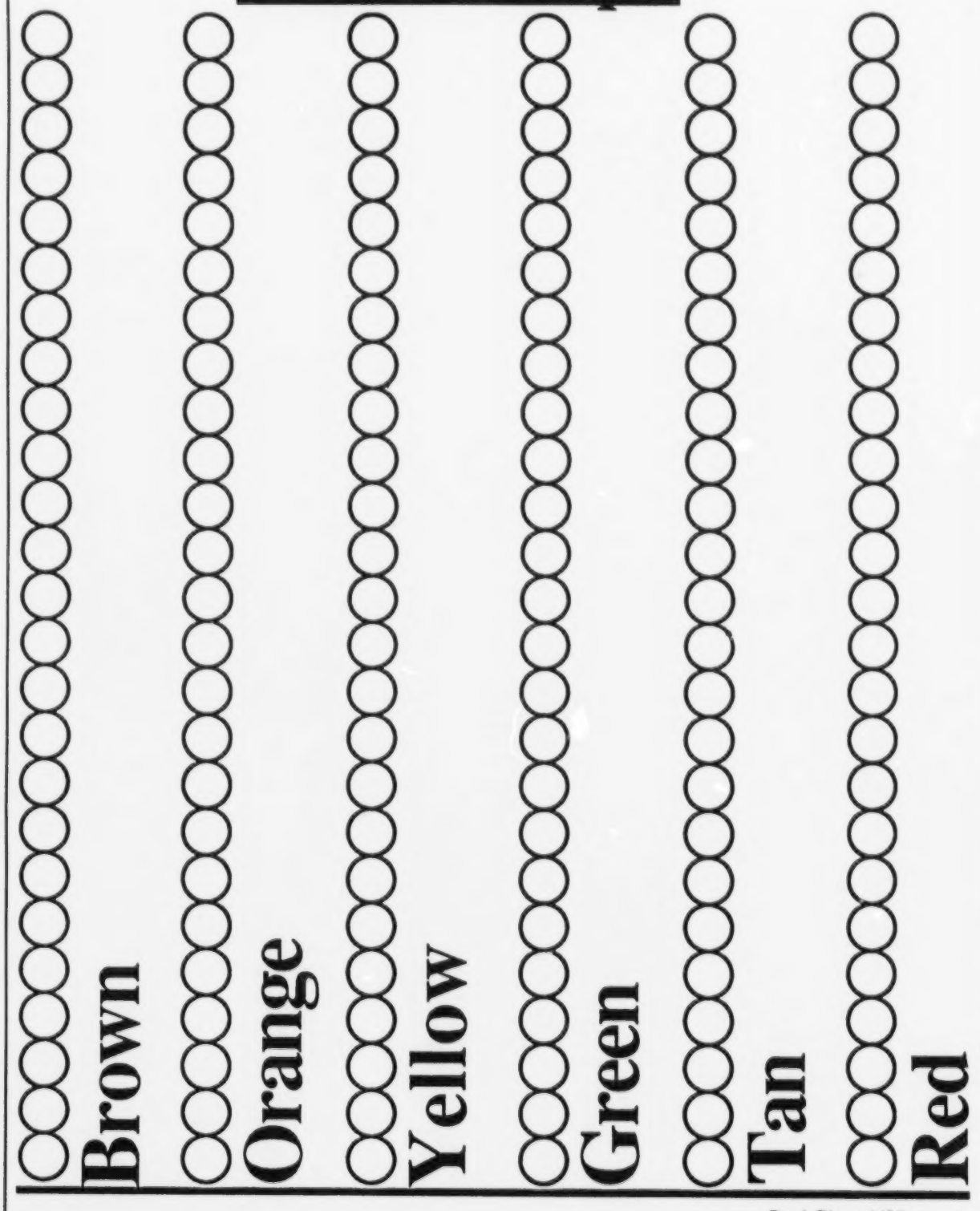
In Professor Bear's Fourth Grade Trophy Room

On they came - Through the tall grass... Twenty-seven angry, relentless, charging fourth graders... And there I was, armed only with a broken piece of chalk, an old eraser, and my dog-eared copy of the textbook!
JN BOYD

The old Professor recalls his many adventures in the fierce Fourth Grade Mathematics Wars.

Name _____

M&M Graph



Paul Giganti '87

22

THIS IS US!
Great Graphs for Kids

By **Delia Sullivan** *and* **Mary Ann O'Neil**

September has arrived, and before you is a list of the names of your new pupils. Once again you are faced with the tasks of quickly learning everyone's name, of helping the children become acquainted with one another, and of easing individuals into their roles as one among others. You are concerned with meeting the children individually, allowing them to be themselves and comfortable with their own self concepts, and yet helping each widen personal horizons as he or she adjusts to being a group member.

There are numerous activities that work toward these goals. Children on the primary level love to read, gather information, organize data, and exhibit their findings. Self-awareness graphs, by depicting the similarities between a child and her or his peers, can be an excellent way of helping children get acquainted in a classroom setting. Mathematics activities free children to think and provide an opportunity for children to discover patterns and relations. Children will enjoy illustrating their individual characteristics as they develop or reinforce the mathematics skills of counting, sorting, grouping, and comparing. There is the added bonus of having instant classroom decorations depicting the most important element of your classroom—the students themselves. The youngsters delight in seeing the traits that they have discovered about themselves and their peers represented in a graphic way.

Starting your mathematics year with a graphing and statistics experience also provides a foundation upon which many ideas can be built.

Delia Sullivan chairs the mathematics department at Oldfield Junior High School in Greenlawn, New York. She also serves as an adjunct assistant professor and guest speaker at Long Island University in the Graduate Education Department. Mary Ann O'Neil is director of the O'Neil Learning Center, an educational facility for the entire family in Newburgh, New York. The Center offers a nursery school, kindergarten through third grade, a dance and circus arts program, and an adult workshop.

Classroom Organization

Primary level

Time allotment

Three, 15-to-20-minute sessions

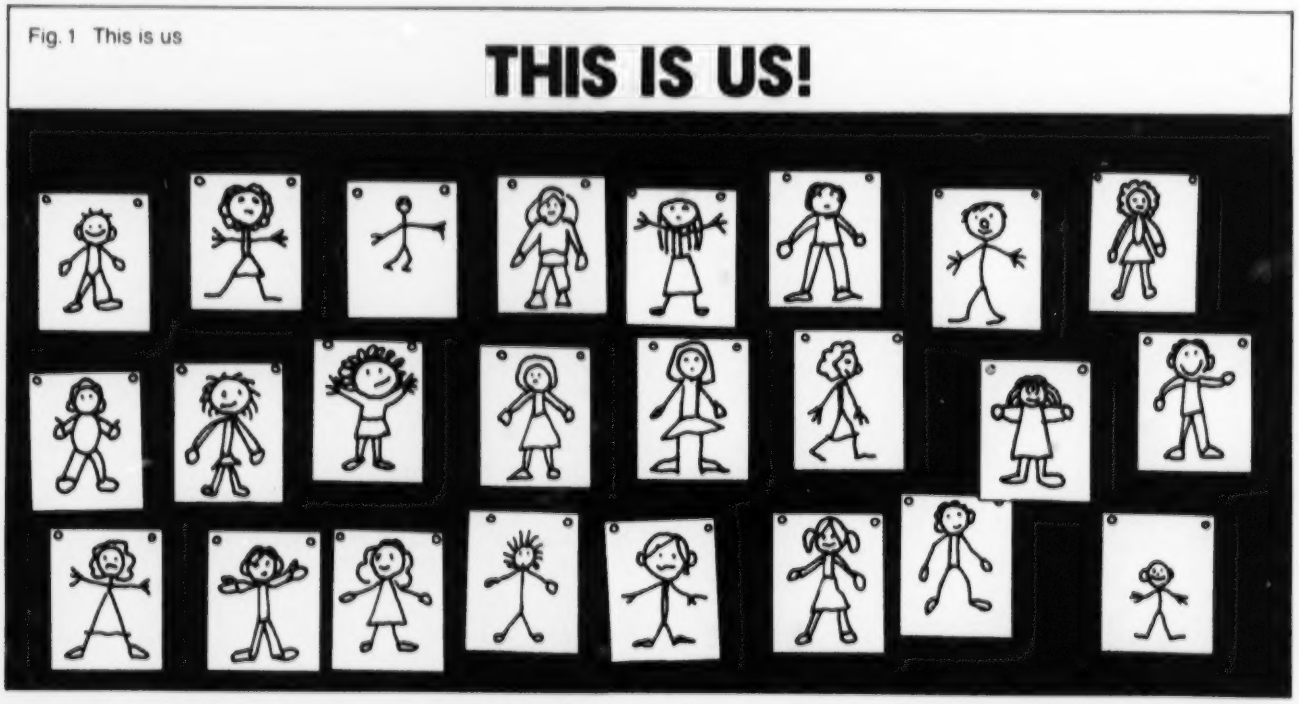
Materials needed

Crayons, tape, one sheet of construction paper for each student (about 10 cm by 15 cm), and one large (about 3 m by 1 m) sheet of white paper.

Set-up

The large sheet of white paper is fixed

Fig. 1 This is us



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to the wall at a level the children can reach.

Day 1. The initial activity, a student-made class portrait, centers around the group as a whole, with no classifications noted. You can open the discussion with the question, If someone asked what everyone in this classroom looked like, how would we show that? At some point, a class picture would be suggested. Allow each child to draw a picture of herself or himself on a small sheet of construction paper. When the children finish their portraits, each will tape loosely her or his drawing anywhere on the large sheet of white paper. (See fig. 1)

A discussion of what they can learn by looking at the large chart would follow. The most easily determined information would be the number of students. This could then be written as an affirmative statement across the chart: WE HAVE 24 STUDENTS IN OUR CLASS.

Day 2. We would like to learn more about our class. The teacher may ask, "How many boys are in our class?" This information is not easily determined from looking at our class portrait. So we remove all the boys' drawings and group them together on our portrait sheet. Boys pictures, and then girls, are taped in horizontal lines, reinforcing the basic left-to-right progression. After the boys and girls are charted in this fashion, a discussion can follow—Which line of pictures is the longer? Are there more boys or girls in our class? Children will begin to see that 16 is more than 8 by looking at which makes a longer line. (See fig. 2)

Day 3. The idea mushrooms and the excitement is contagious. Let's make more graphs! Hair and eye color can also be charted. This time each child is given a happy face and asked to color the hair to match her or his hair color. (See fig. 3) With very young children, a teacher should also put the child's name on each face, to help the children better identify themselves as part of the group. The children put their own hair-color pictures on the chart under the appropriate hair color. To show eye color, children can color small eyes to paste on another chart. (See fig. 4)

Fig. 2 Boys and girls

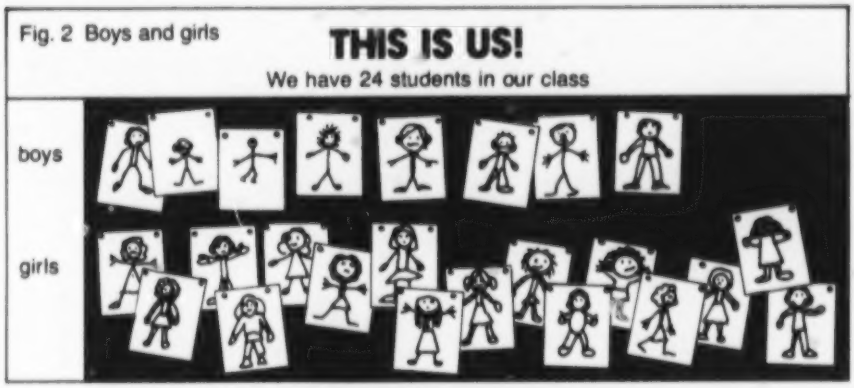

Fig. 3 Hair color

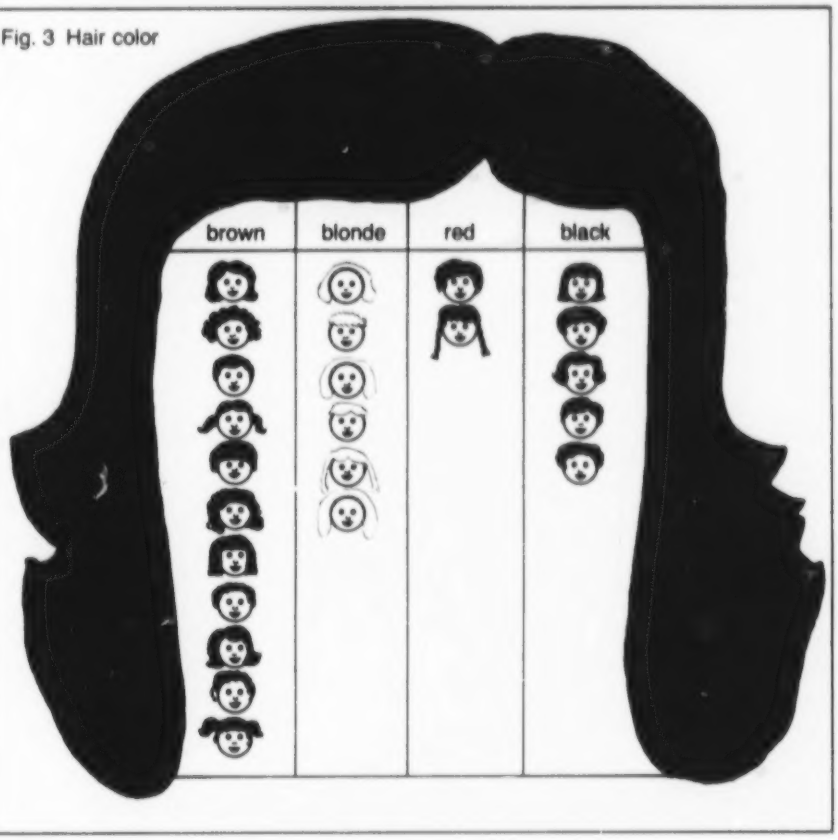

Fig. 4 Eye color

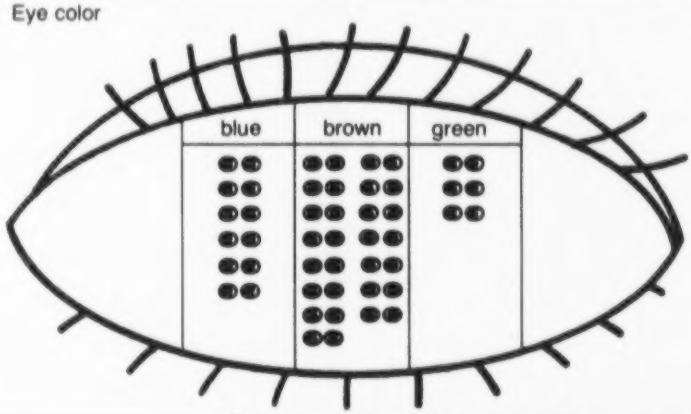

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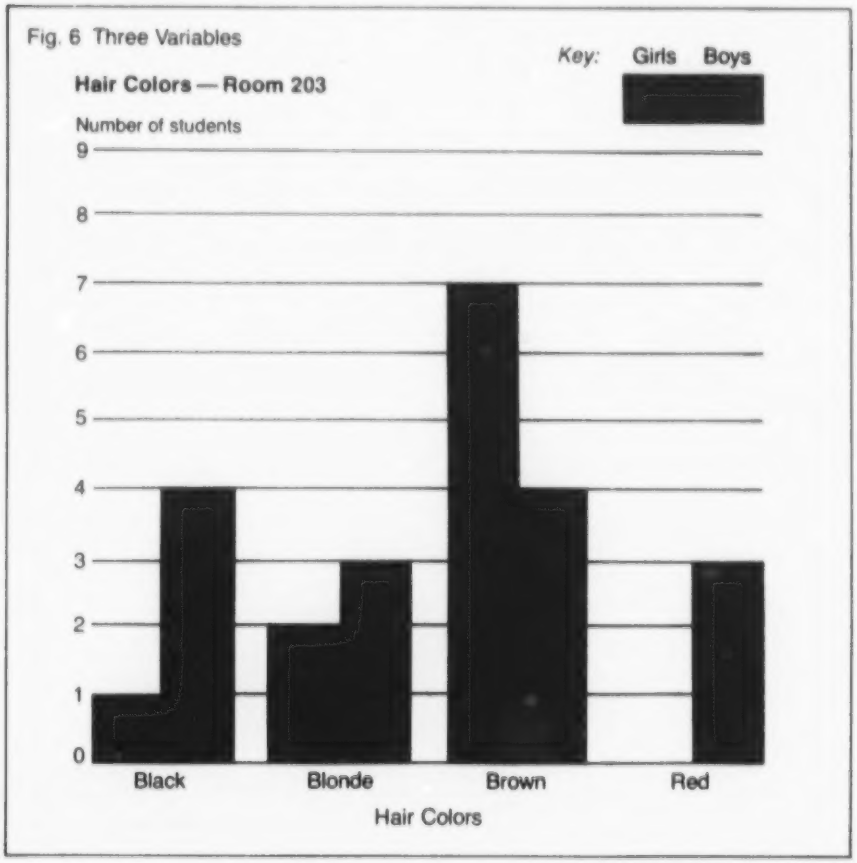
Fig. 5 Two variables

Fig. 6 Three Variables



By looking at these charts, the children begin viewing themselves and their peers as similar and yet safely different. They feel a part of the group as they find themselves on the various charts. They see themselves with a variety of children, showing commonality with different classmates. And they have enjoyed sorting, counting, cooperating, and completing the task with the group.

On another level

Curricula for the intermediate grades include reading and interpreting various types of graphs and, in most instances, making graphs from unorganized data. Some teachers present graphing and statistics as a separate unit, while others interweave the concept throughout various units. The latter can offer a wide variety of learning activities, not only within a mathematics curriculum, but also with social studies, science, and art curricula as well.

The primary-grade activities can be expanded and used at an intermediate level in a more sophisticated form. You

Fig. 7

MATHEMATICS ACTIVITY
Information Sheet

Materials

First day: questionnaire, tape measure, scissors
Second day: graph paper, construction paper, tape, crayons

Procedure

Work with a partner.
Complete the questionnaire as best you can.
Cut the questionnaire along the dashed lines into strips. (You will have six strips, one question on each strip.)
Place each strip in the box in the front of the room that matches the number on the strip.

Fig. 8

QUESTIONNAIRE

1. Your birthday _____ Sex _____

2. Your hair color _____ Sex _____

3. Your eye color _____ Sex _____

4. Your shoe size _____ Sex _____

5. Your height (centimeters) _____

   Your reach (arms outstretched) _____

   Compare your height and reach. Are you a square? _____

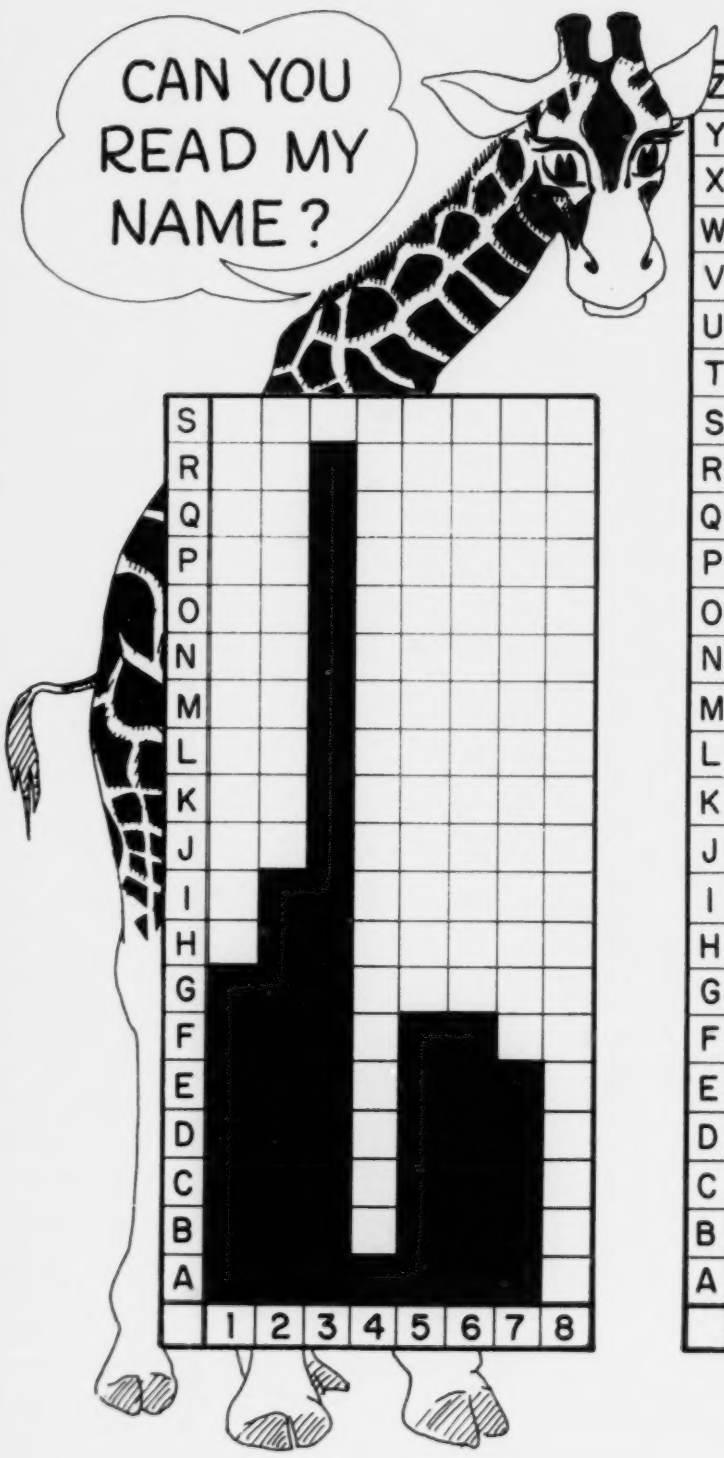
   Sex _____

6. Your favorite sport _____ Sex _____

can move from a teacher-directed activity to one that can become completely or partially student-directed, depending on the capabilities of your class. As the topic is expanded, more personal data can be accumulated—shoe sizes, birthdays, lengths of arm span, favorite sports, favorite colors, lengths of paces, pulse rates, and so on. Your students will probably have ideas of their own.

Once the data is collected, how the data is organized and represented will depend on the individuals involved. Certainly two variables can be included in the graph—for example, month and number as in figure 5—and perhaps three—for example, hair color, number and sex, as in figure 6. If the students have input (and they should) the amount of data they wish to show will dictate the number of variables needed and the best method for representing that data.

IDEAS



GRAPH YOUR NAME

Z Y X W V U T S R Q P O N M L K J I H G F E D C B A

1st 2nd 3rd 4th 5th 6th 7th 8th 9th 10th 11th 12th

Letters in my name

STANDARD 12:
FRACTIONS AND DECIMALS

In grades K–4, the mathematics curriculum should include fractions and decimals so that students can—

- ***develop concepts of fractions, mixed numbers, and decimals;***
- ***develop number sense for fractions and decimals;***
- ***use models to relate fractions to decimals and to find equivalent fractions;***
- ***use models to explore operations on fractions and decimals;***
- ***apply fractions and decimals to problem situations.***

Focus

Fractions and decimals represent a significant extension of children's knowledge about numbers. When children possess a sound understanding of fraction and decimal concepts, they can use this knowledge to describe real-world phenomena and apply it to problems involving measurement, probability, and statistics. An understanding of fractions and decimals broadens students' awareness of the usefulness and power of numbers and extends their knowledge of the number system. It is critical in grades K–4 to develop concepts and relationships that will serve as a foundation for more advanced concepts and skills.

The K–4 instruction should help students understand fractions and decimals, explore their relationship, and build initial concepts about order and equivalence. Because evidence suggests that children construct these ideas slowly, it is crucial that teachers use physical materials, diagrams, and real-world situations in conjunction with ongoing efforts to relate their learning experiences to oral language and symbols. This K–4 emphasis on basic ideas will reduce the amount of time currently spent in the upper grades in correcting students' misconceptions and procedural difficulties.

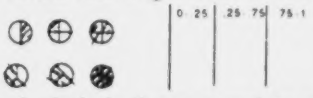
STANDARD 12:

- **use problem-solving approaches to investigate and understand mathematical content;**
- **formulate problems from situations within and outside mathematics;**
- **develop and apply a variety of strategies to solve problems, with emphasis on multistep and nonroutine problems;**
- **verify and interpret results with respect to the original problem situation;**
- **generalize solutions and strategies to new problem situations;**
- **acquire confidence in using mathematics meaningfully.**

Activities for the Fractions and Decimals Standard

1. Divide the class into groups of 2, 3, 4, 5 and 6's. Give each group a candy bar or other type of food. The groups have to divide the candy into equal pieces for each member. The teacher can move from group to group to see that they see that 1/5, 2/5, 3/5, 4/5, 5/5 is equal to 1 whole for example. (Sheri Wilcox)

2. Have a velcro learning center in room



place each circle in the appropriate column and then have a folder in the desk with the answers and an explanation for each answer. (Laura West)

3. I would use the activity "Rolling Fractions." In this activity, the students have a game sheet and two dice labeled with fractions. The student rolls the dice and shades in the greater of the fractions. The object is to shade in all the squares. Also on the dice. I would put the decimal equivalencies (1/2 = 0.5) so that students recognize they are the same amount. (Jennifer Hoelk)

4. After making pumpkin pies for Halloween put 1 pie at each table. Have the children decide how many pieces would be needed to get everyone a piece. As the teacher cuts the pie use fractional language. "So you say we need 8 pieces of pie, so I'll cut the into into 1/8th each of you will get 1/8 of the pie." (Deena Loan)

5. Use a game for two or more players, like Old Maid fraction cards and card with shaded regions. Match the fractions to the regions. (Donna Davis)

These ideas are from Tech students in the K-8 certification program.

TEACHING THE CONCEPT OF $\frac{1}{2}$ IN THE PRIMARY GRADES

By **Ann Miller**

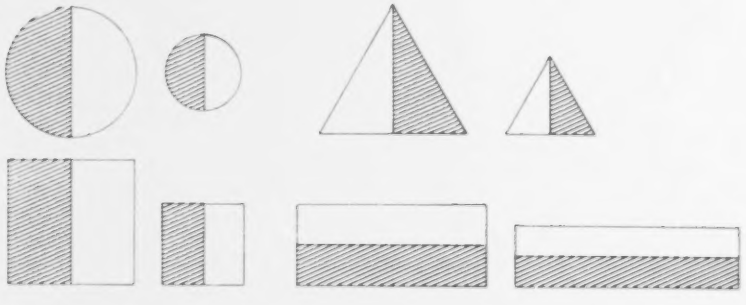
A fundamental concept introduced in the primary grades is the number one-half. It is of particular importance for two major reasons. First, it is the student's first experience with a fractional number, and hence a thorough understanding of this concept will facilitate the introduction of other rational numbers. Secondly, most children entering school have some preconception of *one-half* and often these basic notions are imprecise, if not erroneous. This is evidenced by the all too frequent use of such phrases as "the bigger half" and "the smaller half." Thus it is important that the student clarifies the concept of one-half in the primary grades and possesses a clear and correct understanding of this idea. The following are suggested approaches and activities designed for the primary grades to introduce and reinforce this concept.

It should be noted that there are two fundamental uses of the concept, namely, *one-half of the number* and *one-half of the size*. Classroom time should be devoted to both of these considerations. *One-half of a number* can be introduced through exercises and games in which the student is asked to find one-half of a set of two clowns, or of six automobiles, or of a dozen eggs. This type of exercise occurs frequently in textbooks and helps to familiarize the student with the idea of one-half of a number of discrete objects. However, classroom activities are an excellent reinforcement and should not be omitted. A simple example that the students are familiar with is choosing sides; a class with an even number of students can divide itself into two halves with each side representing one-half of the class. Questions such as, "Is the set of girls in the class one-half of the class?" and "Is the set of red-headed students one-half of the class?" are other possible considerations. These are useful since they generally tend to be counter-examples, thereby illustrating that dividing the class into two parts is not identical with dividing the class into two equal parts. Other activities can be performed that require the students to find one-half of a number of objects, such as sorting out one-half of the number of crayons in a box or writing one-half of the letters of the alphabet. This latter activity has several derived benefits. The majority of students will, on their first attempt, merely write the first thirteen letters of the alphabet. They should be encouraged to find other sets that are one-half. One particular decomposition should be discussed; namely, the set of letters formed by beginning with the letter *A* and writing every other letter of the alphabet. First, this procedure illustrates the simplest example of skip counting and informally suggests the relationship between one-half and dividing by two. Secondly, if the complementary set of letters is written alongside the original set, the students can see the one-to-one correspondence that exists between the two halves of a set.

Fig. 1

To introduce the concept of one-half of the size, most textbooks use partially shaded diagrams. It is desirable to begin with classroom activities prior to the use of visual diagrams. Although there are many excellent ready-made materials on the market made of plastic or cardboard, students in the primary grades enjoy and benefit from making their own models out of string and construction paper during activity sessions. In addition to fostering student involvement, the models are less abstract than drawings. Thus, the student who has folded a length of string into two pieces of equal length and a sheet of paper into two equal parts will better comprehend the meaning of a sub-

Fig. 2



A former elementary and junior high school teacher, Ann Miller has been involved in teaching mathematics education courses for the past five years at the university level. She is on the faculty of the Department of Mathematics of Southern Illinois University, Carbondale, Illinois.

divided number line or a shaded rectangle in the textbook.

Diagrams typical of those found in textbooks are given in figure 1. Although these drawings have many good aspects, they suffer from overexposure and oversimplification. First of all, figures that are similar but not congruent should also be given. This emphasizes that it is not the size of the shaded region that is important, but rather the size of the shaded region in relation to the whole region. Some examples of this are shown in figure 2.

Students should also be aware that a model can be separated into two halves by various methods. Thus, a figure should be given with several such shadings, as in figure 3. Students should also be encouraged to find other such decompositions.

Another benefit of the use of physical models is that they provide a means of verification that each of two portions is, in fact, one-half of the original. This is of significance since it presents the concept of congruence by the readily understood method of superposition. The use of physical models helps the student grasp the fallacy inherent in the often-used expression "the bigger-half." Folding physical models can lead the student to realize that the two halves of the whole must be equal in size, and hence that neither region can be "the bigger half." Thus, visual diagrams should also be given that reinforce the distinction between the phrases *one part of two parts* and *one part of two equal parts*. These are illustrated in figure 4.

Representational drawings and physical models dealing with measurement not only illustrate the concept of one-half, but also introduce the student to various units of length and weight. Thus, students can join the ends of a meter-long string and cut in the center to produce two lengths of string, each one-half of a meter long; or pour a kilogram of rice onto the two sides of a balance scale until they have two equal portions, each of which is one-half of a kilogram.

As the student considers more and varied examples, he can be introduced to more sophisticated drawings and models. We can and should employ different shadings with multiple divisions. Some examples are given in figure 5.

Fig. 3

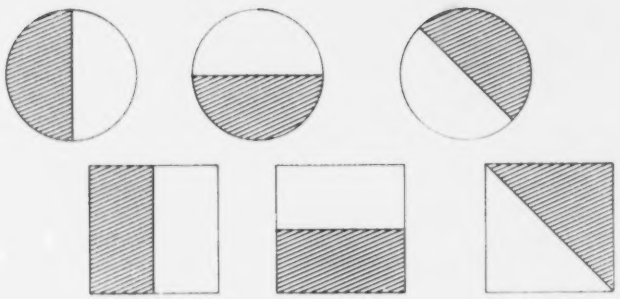
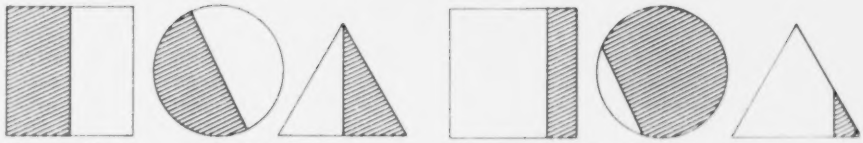


Fig. 4



One part of two equal parts

One part of two parts

Fig. 5

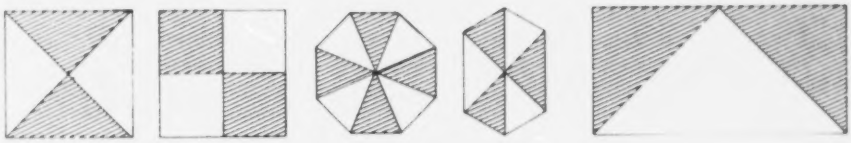
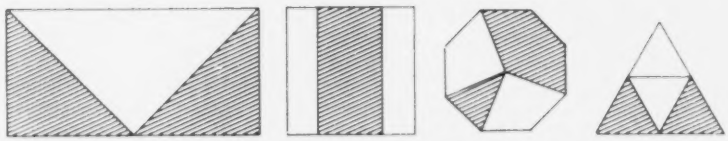


Fig. 6



Given such drawings, the student can trace the pattern and construct a replica. The student can then verify that the shaded portion is one-half of the entire figure by cutting out the shaded regions and superimposing them onto the unshaded portions. Not only does this approach extend the concept of one-half, but it also prepares the student for the notion of equivalent fractions. Furthermore, the drawings illustrated in figure 6 help the student to realize that the number of apparent subdivisions is irrelevant; the crucial fact is the relative sizes of shaded areas to unshaded areas. Again, the student can check each of these by constructing traced replicas and superimposing the shaded region onto the unshaded portion.

Lastly, once the student has a working knowledge of one-half and the halving process, we can proceed to the idea that doubling is the opposite process to halving. Here again physical models can play an important role in the learning process. This prepares the student for the fact that *one-half of* is equivalent to the concept of *dividing by two* and that these operations are inverse to *multiplying by two*.

Of course, the classroom teacher must decide how much time should be devoted to the concept of one-half. However, rapid learners who quickly grasp the idea of one-half will be challenged by the more sophisticated representations and the slow learners will have numerous examples to provide positive reinforcement.□

STANDARD 13:
PATTERNS AND RELATIONSHIPS

In grades K–4, the mathematics curriculum should include the study of patterns and relationships so that student can—

- ***recognize, describe, extend, and create a wide variety of patterns;***
- ***represent and describe mathematical relationships;***
- ***explore the use of variables and open sentences to express relationships.***

Focus

Patterns are everywhere. Children who are encouraged to look for patterns and to express them mathematically begin to understand how mathematics applies to the world in which they live. Identifying and working with a wide variety of patterns help children to develop the ability to classify and organize information. Relating patterns in numbers, geometry, and measurement helps them understand connections among mathematical topics. Such connections foster the kind of mathematical thinking that serves as a foundation for the more abstract ideas studied in later grades.

From the earliest grades, the curriculum should give students opportunities to focus on regularities in events, shapes, designs, and sets of numbers. Children should begin to see that regularity is the essence of mathematics. The idea of a functional relationship can be intuitively developed through observations of regularity and work with generalizable patterns.

Physical materials and pictorial displays should be used to help children recognize and create patterns and relationships. Observing varied representations of the same pattern helps children identify its properties. The use of letters and other symbols in generalizing descriptions of these properties prepares children to use variables in the future. This experience builds readiness for a generalized view of mathematics and the later study of algebra.

Patterns and Relationships in the K-4 Curriculum

Bob Underhill

Last spring (1989), I was contacted by Ruth Sheffield down at Washington County. She asked me if I would conduct a workshop for primary teachers at the fall meeting of the Southwest Virginia Council of Teachers of Mathematics at Emory and Henry College in the fall. I agreed to do so and set about reflecting how what I already know is related to the Patterns and Relationships Standard. Further, I started reflecting about the various dimensions of the topic. I quickly settled upon the following dimensions which, while they expanded in richness and complexity, remained my main categories. I settled upon the following ways to think about patterns and relationships in K-4:

1) Related to the development of *number* concepts;
2) Related to *geometry* concepts, and
3) Related to *measurement* concepts.

Upon further reflections, it occurred to me that the richness came in the combinations which could be further derived. Specifically, there are also patterns and relationships related to the merging of:

4) Number and geometry concepts;
5) Number and measurement concepts;
6) Geometry and measurement concepts, and to the merging of
7) Number, geometry and measurement.

I'd like to write a long piece on these relationships as we talked about them for an hour. However, I'm on sabbatical this semester with a whole host of projects calling for my attention so I'm going to handle this as a teaser; I'll give you some notion about where you can go and you can draw upon your own experience and education to extend the ideas.

First and foremost was an insight which struck me a couple years ago when I was working with the kindergarten teachers in Carroll County. The concept of classification is a generic activity in human learning. When the young child learns the concept of "ball," he or she has learned to sort out all the things in the environment into those things which are balls and those things which are not. After classifying by nouns (persons, places and things, as I recall from my seventh grade English teacher!), the child learns also to classify by verb (rolls, bounces, rattles, etc.). Extend this notion and you get more and more abstract: sounds, feels, olfactory and on to relationships "is the Mother of," "is older than." The concepts get further and further removed from sensory qualities until you get to very abstract concepts: truth, justice, four. Four? Yes, four is *very* abstract. It is a quality of many things and yet it is not a quality of anything. There may be four marbles, but fourness is not an attribute of the marbles, only of their "manyness." All of this is simply the route to our first dimension, number.

Number

At the pre-number level, we engage learners in activities which develop both their classification and relation skills *and* make them more aware of their classification powers. We begin with sets of objects which are highly sensory which they can sort by shape, color, smell, and so on. Then we can extend these powers to *function* (things we eat with, write with, wear, and so on). We can do these sortings with one attribute and later by two attributes (red which we wear; metal things we eat with, and so on).

Another pre-number pattern emerges from matching sets and creating one-to-one correspondences. By learning to set the table, button a sweater, pass out pencils and so on, learners are developing the ability to create sets of three, four and five by building sets which are in one-to-one correspondence with given sets. These skills can be used pre-numerically for establishing which sets have more, less and the same. All of these things are so obvious to us. They comprise the foundations of meaningful mathematics learning for young children.

When children begin to acquire basic number concepts, the patterns and relationships among numbers can be explored. We see these emerging through the one-more-than and one-less-than concepts. Further elaborations are possible through counting by 1s and counting by 10s. Interesting patterns can be observed by starting a 100s chart, a calendar, working with Cuisenaire Rods, skip counting, exploring the meaning of odd and even in rectangular block arrangements and so on.

Geometry

Just as we could explore number without measurement and geometry, so, too, can we explore geometry by itself. The types of experiences which will lead to the discovery of geometric patterns and relationships are these:

1) Block stringing (cube/sphere) first with two shapes, then with three;
2) Children can work in pairs to find things in the room with certain shapes;
3) Repeat (2), above, with old newspapers and magazines;
4) Classify shapes by geometric characteristics (curvy, pointed, flat, etc.);
5) Creating matching shapes on a geoboard, and
6) Creating designs with a given shape or set of shapes.

Measurement

Often when we think of measurement we automatically think of number; however, measurement concepts do, indeed, have non-numerical bases.

1) Create real situations which require length comparisons without number (e.g., will this table fit through the door? Will this bookshelf fit under the window?). One of the great learning experiences in solving such problems is children's *creation of solution strategies*. They will often think of ways to answer questions which didn't occur to us!
2) Length seriation - short, medium, long, three people, three books on the shelf, etc.
3) Area - which table will permit us to set up a bigger display? Do I have enough tagboard to cover the bulletin board?
4) Area seriation: three sizes of book *covers*, three

tables, three lakes on a map, tower of Hanoi puzzle.
5) Volume - which aquarium will have more space for our fish? Which of these jars or cans will hold the most sugar?
6) Volume seriation - three containers of small, medium and large interiors - Monkey-in-a-barrel; Russian peasant seriation puzzle, measuring cups, mixing bowls.

Number and Measurement

Basically, this dimension is achieved in three stages:
1) Use arbitrary units to turn length, width and volume into countable quantities.
2) Use standard English and metric units, and
3) Explore relationships among units, (e.g., 1 meter = 100 centimeters).

Number and Geometry

This dimension can be achieved through several routes:
1) Add numerical criteria to block stringing;
2) Art with certain specifications (e.g., create a picture with 3 circles and 2 triangles);
3) Counting shapes;
4) Counting elements of shapes with certain attributes (e.g., how many sides has this figure?).
5) Creating shapes with numerical specifications (e.g., make a figure on the geoboard with five sides; with 7 pegs in its interior).

Geometry and Measurement

1) Shapes which are longer/shorter; bigger/smaller (area); bigger/smaller (volume).
2) Seriate parts by length or by area or by volume.

Geometry, Measurement and Number

Length: Use sides of figures as iterable units to measure length.
Area: Use squares, rectangles, etc. as iterable units to measure area.
Volume: Use cubes or other shapes as iterable units to measure volume.

What all this proves is that there are seemingly endless opportunities and relationships and patterns which can be explored by young learners. Happy exploring!

Patterns in Developmental Growth

Than Than Van

The new standards describe mathematics as the study of patterns. In this short article, I will use simple classifications to describe how children must use patterns to classify things which are sensory laden and, over time, be able to strip concepts of their sensory components to end up with abstractions called numbers.

Piaget's work and that of many other researchers, has indicated that classifying is a prerequisite concept to understanding number concepts. So, teachers need to provide appropriate learning experiences to build solid *readiness foundations* which facilitate acquisition of number concepts.

Number is a *quantitative characteristic (attribute) of a collection,* and counting is a strategy for determining this quantitative characteristic. Counting has both *ordinal and cardinal components*; therefore, classification and seriation are important readiness foundations for number concepts.

Since the child's learning style is very physical, teachers can provide opportunities to create sets by simple classification of the *qualitative* characteristic of objects, e.g. blue-not blue. Teachers can provide classification experiences with qualitatively discernible attributes such as *size, shape, color, hue, texture* and *composition which can be detected by using senses of touch, taste, smell, hearing,* and so on. This classification experience should move away from immediate sensory experience to more abstract notions. By doing this the child becomes aware that classification can occur with *any sense* and the ultimate step is to classify with *no senses.*

The next step is providing seriation activities (putting elements in proper *order and making patterns*), e.g., children seriate objects that vary in degree by one attribute, e.g., *tall, taller, tallest* (seriation by height, or, red, redder, reddest). These pre-number seriation activities provide readiness for number seriation (e.g., 4, 5, 6).

After many qualitative classification and seriation experiences both planned and incidental, children learn to classify sets quantitatively. First, using gross quantitative classification, such as all, some, more than, less than, and the same. Next, use one to one correspondence with two sets of objects and build one-to-one correspondence and the sense of sameness or "equality" which is pre-requisite for developing understanding of relation through matching.

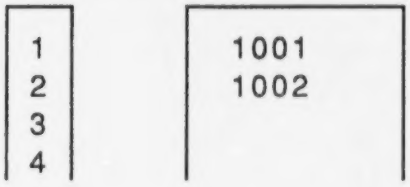
After a child has concepts of classification, seriation and one-to-one correspondence relationship, they can develop counting which involves one-to-one correspondences between the counting sequence and some set of objects. And finally, twoness (a property of sets having two elements) are related to a set, a standard numeral and a word name.

Than Than is from Burma. She is engaged in graduate studies at Tech.

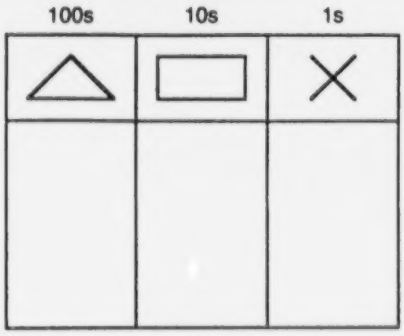
Some More Pattern and Relationship Ideas

These are some ideas which emerged from the small group discussion/brainstorming sessions we had at the workshop described in the previous article:

1. Use a 100's chart for money or counting using 10's.
2. Compare sizes of birthday present boxes. Long - short ---both boxes same size in volume.
3. *Place Value*. Use base blocks. Start with 1. Then so 2, etc. Have children write the number on adding machine tape or strips of paper. Go as far as you want, but show trading 10 ones for 1 ten and ten tens for 100. Check childrens writing as they go along to see that they can write.



4. Creating geometric shape books. Each page uses different geometric shape - students make patterns or pictures.
5. Fold a paper in half - use a string (that has been dipped in paint) to form a closed or open design on paper - press on folded paper - pull string to form design - use creative thinking to complete the picture - i.e. butterfly, spaceship, etc.
6. Use line of symmetry concept - have students fold paper and draw design on folded side - cut out design - students label line of symmetry - then use imagination to make a bug - give bug a name and create short stories with symmetrical insects.
7. Bundles of toothpicks for place-value.
8. Color ice cream sticks to represent ones and tens.
9. Exchange Game



Roll dice, place no. of ones in ones column. When get ten in ones column must change for one ten. First one to get established number of triangles is the winner.

10. *Volume*. Wet sand and put in various containers and dump to compare volumes in addition to shapes.
11. Yard is used to measure length.
12. Flannel board - fractions.
13. Pie plates - fractions - pizza parts.
14. Less than, greater than - Alligator head-mouth opened one direction and the other. Point out that the Alligator will always eat up the larger number.
15. Folding paper to do fractions. Ask the child to fold in ½'s, ¼'s, etc. Let them cut them out. Paper patterns different shapes.
16. Use hundred's chart to color multiples.
17. Use fraction bars, fraction pie pieces to find meaning of fractions by discovery and through games.
18. Cut symmetrical pictures from magazines in half and have the children draw other half.
19. Cuisenaire Rods for seriation - length.
20. In teaching place values: Teachers use blocks to teach tens-ones. Have students draw pictures to show numbers such as 23.
    Masking tape around popsicle sticks to show tens - ones.
21. Lots of "hands-on" practice to depict number concept before doing computation.
22. Art work involving geometric shapes - ask "What can you make with the shapes you have?"

Patterns: Growth in Abstraction

Bob Underhill

This is the story of simple classification experiences and how it moves from *qualitative* attributes to *quantitative* attributes. These are simple "yes" "no" classifications. Here is an *example* of a sequence from simple to more complex and more abstract.

A. *Highly sensory and experience based.* (learner DOES these) (concrete)
   - visual: chair/not chair
   - auditory: sound of a car/not the sound of a car
   - olfactory: chocolate/not chocolate
   - tactile: soft toy/not a soft toy
   - kinesthetic: toy that bounces/toy that does not bounce
   - taste: sweet cereal/not sweet cereal

B. *Highly sensory* and not as experience based (DO these) (more abstract)
   - visual: triangle/not triangle
   - auditory: high pitch/not high pitch
   - olfactory: fragrant/not fragrant
   - tactile: hard/not hard
   - kinesthetic: flat/not flat
   - taste: salty powder/not salty powder

C. *Highly experience based* and not as highly sensory. (Pictures and memory)
   - visual: boat/not boat
   - auditory: quiet/not quiet
   - olfactory: like the smell/don't like the smell
   - tactile: wet/not wet
   - kinesthetic: tiring/not tiring
   - taste: yukky/not yukky

D. *Not* highly experienced based and *not* highly sensory. (Pictures and diagrams)
   - visual: square/not square
   - auditory: notes printed on a scale
   - olfactory: scratch and smell (no pictures or objects)
   - tactile: (temperatures stated) cold/not cold
   - kinesthetic: tell me something which is round/not round

E. *Qualitative* but *abstract*. (Recognition, language)
   - visual: which of these words have a K in them?
   - auditory: which of these words have the "ch" sound?
   - olfactory: which of these words name things which smell good?
   - tactile: which of these words name things which are spongy?
   - kinesthetic: which of these words name activities which make your legs work hard?
   - taste: which of these words name salty things?

F. *Most abstract, qualitatively*. (Recall)
   - visual: name something with three wheels
   - auditory: name an instrument with a very high pitch
   - olfactory: name something which smells like home
   - tactile: name something which feels scratchy
   - kinesthetic: name an activity which is very tiring
   - taste: name something which is very sour

With all this successive abstraction, we are probably ready now to deal with sets which are in one to one correspondence and those which are not. THEN, we are ready to use 1-1 correspondence to talk about those sets with two somethings and those without.

Communications concerning the New Standards
Is really picking up!

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Variations on Rummy

By **Rebecca S. Nelson**

Rummy is a sufficiently interesting card game for even adults to spend leisure time in its pursuit. Because of this high interest in the game, variations on rummy can provide many hours of excellent practice for related mathematical concepts at both the pictorial and abstract levels. This practice can be offered without requiring any paper grading by the teacher and, since only three to five players are needed to play rummy, the wise teacher may provide for individual differences by careful assignment of students to the appropriate game.

General rules as modified for children. The dealer deals five cards to each player and turns the remaining cards face down on a "draw stack." The top card of the draw stack is turned up to begin a discard pile. The first player may draw either the top card on the draw stack or the card from the discard pile. Subsequent players may take one card from the draw stack or up to four cards from the discard pile. If the player wants the third card in the discard pile, he must also take the first and second card. After each play, the player must discard one card to the discard pile. The object of the game is to get rid of all the cards in your hand by (1) making four-card books, (2) making three-card spreads, (3) playing a fourth card on another person's three-card spread, or (4) discarding. Play continues until one person is able to get rid of all the cards in his hand. If the draw stack becomes depleted, the discard pile is shuffled and placed face down to be the new draw stack.

Materials. Each rummy deck requires fifty-two, three-by-five cards to make thirteen books with four cards in

Fig. 1. Money Rummy

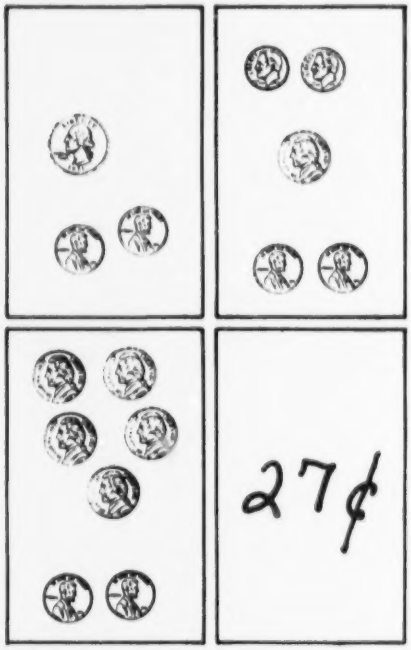


Fig. 2. Place Value Rummy

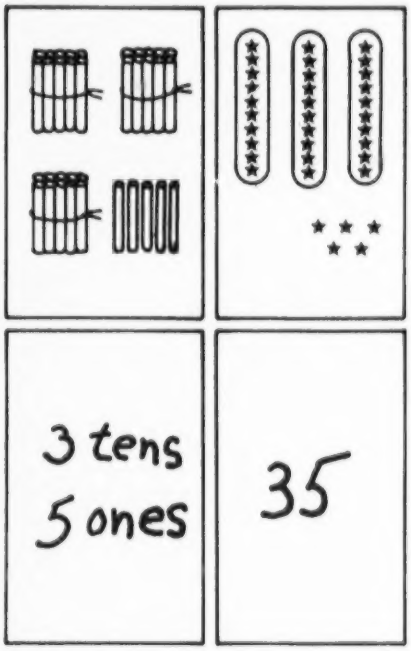


Fig. 3. Mixed Rummy

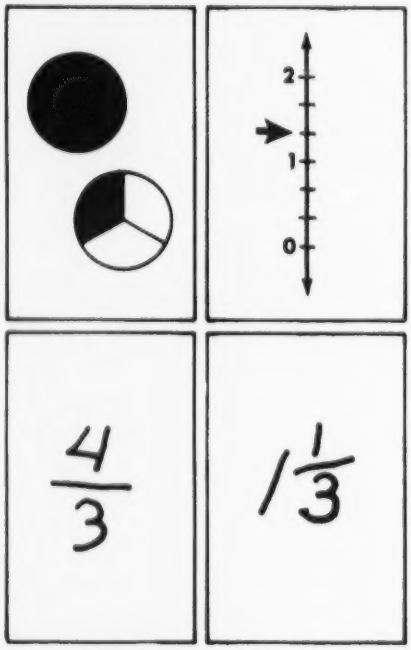
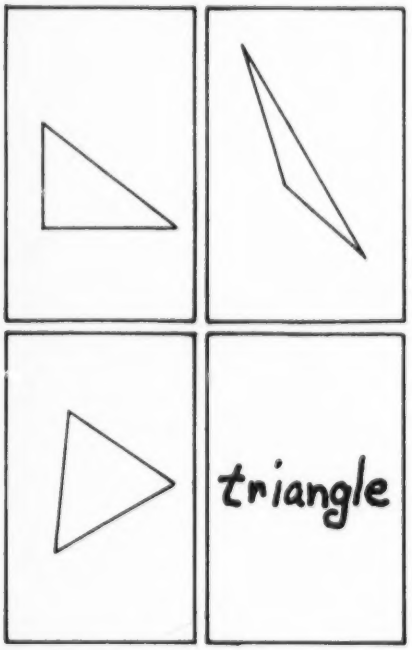


Fig. 4. Shape Rummy



An associate professor of mathematical sciences at Ball State University, Rebecca Nelson works with preservice and inservice teachers in content and methods courses.

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each book. A book is a set of related facts or pictures. Rummy cards can be made on many different topics. The sample books shown here suggest a few of the possible topics.

Besides the fun involved, low cost factor, elimination of needed paper grading, help for individualized differences, and plenty of practice available, rummy offers the extra bonus of having a chance factor. Because the winner is determined by the shuffle and draw of the card, the brightest child does not always win, and the not-so-bright child does not become as easily discouraged.□

Fig. 5. Related Facts Rummy (Multiplication, Division)

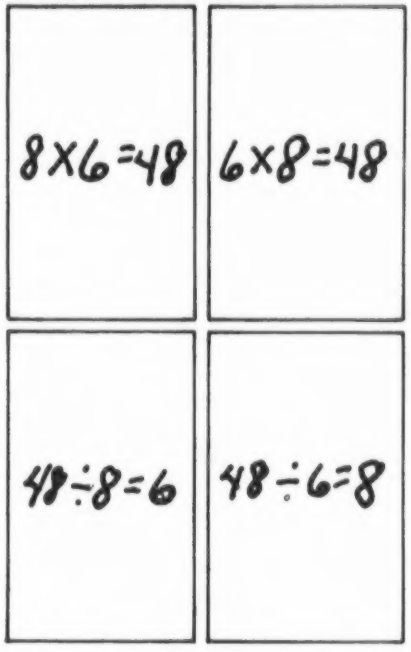


Fig. 6. Family of Fractions Rummy (Ratio)

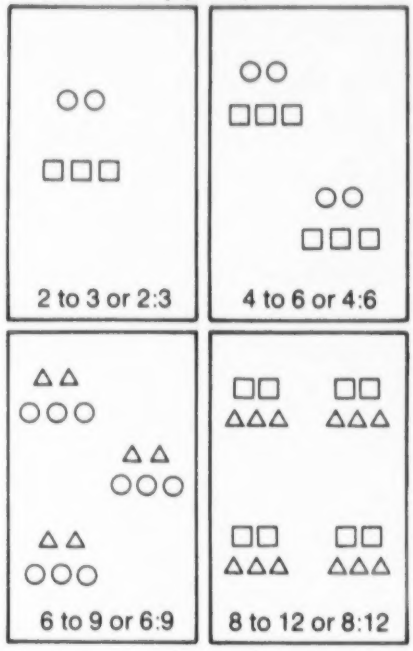


Fig. 7. Sum Rummy

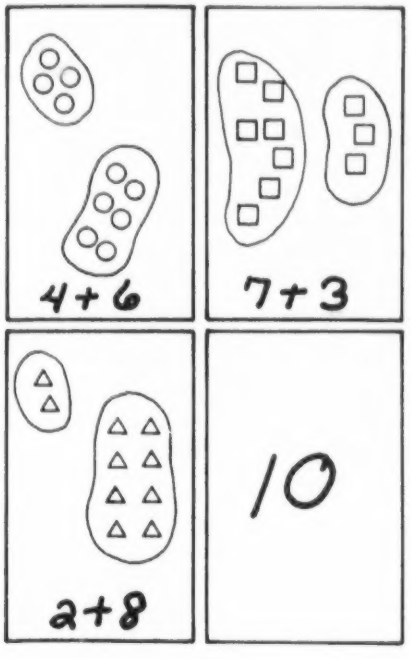
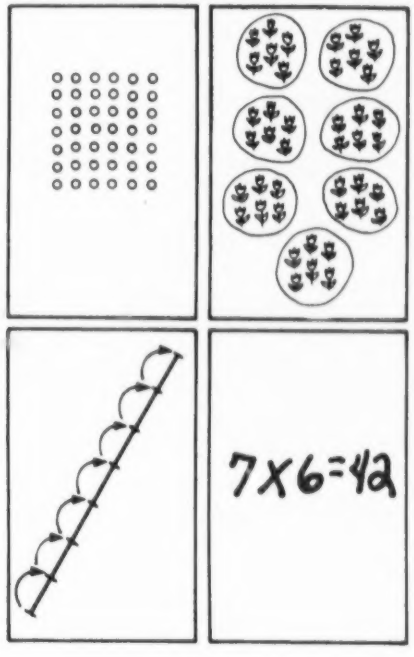


Fig. 8. Multiplication Rummy



Let's Do It

Food for Thought

By **Marcia E. Dana**
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Madison, Wisconsin
and **Mary Montgomery Lindquist**
National College of Education
Evanston, Illinois

Before Thoughts

Have you been thinking that you should do something different? Something practical? Or something that involves and interests the children in doing some mathematics? Here's some food for thought—for third graders, for fourth graders, . . . , all the way up to eighth graders. Take a look at questions that came quickly to our minds as we read the Best Burger menu.

1. You're having lunch at Best Burgers. List what you are ordering. Find the cost of your order.
2. How much more is a cheeseburger than a hamburger?
3. List all the sandwiches that cost over 75¢.

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4. If you bought 6 Greatburgers, how much would they cost?

5. How many small orders of onion rings could you buy with $2?

6. If you bought a cheeseburger and a medium-sized shake and paid a dollar, how much change would you get?

7. Which of these orders can you buy with a dollar?

(*a*) Hamburger, large fries, small cola

(*b*) Cheeseburger, small fries, medium cola

(*c*) Hamburger, small onion rings, milk

8. Best Burgers had only these things left one day:

Great Burgers, Chambos, and Seaburgers

Small colas, medium colas, and large colas

If you wanted to order one sandwich and one drink, what different combinations could you order?

9. What's the most expensive thing on the menu?

10. What is the favorite sandwich and drink of each child in your class? Make 2 graphs to show the results for your whole class.

In-Between Thoughts

Now that you have thought some about this menu, let's consider reasons for using menus. Since almost everyone (children included) at some time uses a menu, it gives an opportunity to apply mathematics to real situations. Children are often interested in choosing foods, or they may detest certain foods. In either case, problems about menus can be posed in a fun and motivating way.

Menus are often easily obtainable from your own community. If the menus themselves are not available, you or the children can copy items and prices from menus. Once you have at least one menu, you and the children can be very creative in generating a variety of problems. The problems can be very straightforward or quite open-ended, depending on the needs of your class.

In the sections that follow, we reexamine the Best Burgers menu in terms of suggestions for practice and for different problem-solving ideas. These are the topics that helped us generate questions about the menu and we hope they will give you ideas for creating many more menu problems. However, don't kill the idea. We have included here more than any one class would need to do. Choose and use the ideas as long as interest holds with your class.

Practice

All the suggestions in this section are designed to provide quick simple practice. They should involve a minimum of preparation on your part and a maximum of meaningful practice on the children's part.

Ordering numbers

There are many ways to practice ordering numbers on this menu. Here are a few.

BEST BURGERS
way to go!

Menu

Greatburger	.89
with cheese	1.04
Great Greatburger	1.45
with cheese	1.60
Littleburger	.65
with cheese	.72
Hamburger	.35
Cheeseburger	.42
Double hamburger	.70
with cheese	.84
Chambo (cheese/ham)	1.19
Seaburger (fish)	.89
with cheese	1.04
French fries	.32 and .47
Onion Rings	.42 and .59
Apple pie	.35
Soft drinks (cola, orange)	
Small	.25
Medium	.30
Large	.40
Shakes (chocolate, vanilla, strawberry)	
Small	.40
Medium	.55
Large	.80
Coffee, tea	.25
Milk	.25
In season:	
Hot chocolate	.30
Iced tea	.30

Prices subject to change without notice.

1. Find the cheapest and the most expensive sandwich (or drink) on the menu.

2. Write all the drinks (or sandwiches) in order, from cheapest to most expensive.

3. List all the things on the menu that are over (or under) a given price.

Addition

1. To practice adding two numbers, you could make a list of pairs of sandwiches and drinks and the children could find the total for each pair.

	Total
1. Cheeseburger and milk	
2. Greatburger and small cola	
3. Seaburger and large orange	

Instead of listing these yourself, you could ask each child to make up a list of 10 pairs and find the total for each pair.

2. To practice adding more than two numbers, a child could make up an order for one or more persons and find the cost of that order.

Subtraction

1. One kind of subtraction problem involves how much more (or less) one thing on the menu costs than another. You could make up a page of such questions:

	How much more?
1. Greatburger than a cheeseburger	
2. Large cola than a small cola	
3. Chambo than a Seaburger	

Again, the children could make up their own pairs and find the difference for each pair.

2. Another kind of subtraction problem involves making change. You could ask the children to list each of the things under a dollar on the menu and tell how much change from a dollar they would get for each thing. Or you could use another amount, such as 75¢.

Multiplication

1. To practice multiplying by a one-digit number, you could pose problems such as these:

Pretend that you and two friends are ordering each of the following items. List how much each would cost for the three of you.

	3 of them
1. Small onion rings	
2. Big strawberry shakes	
3. Chambos	

Or you could pose sets of separate problems:

How much would each of these cost?

1. 7 cups of coffee
2. 9 hamburgers
3. 5 large orders of fries

2. To practice multiplying by a two-digit number, you could ask the children to figure out how much it would cost if everyone in the class ordered a Greatburger, with cheese or a medium chocolate shake, or small fries. (Any other item on the menu could be used as well.) Or, of course, you could pose sets of separate problems: How much would 42 cheeseburgers cost? How much would 12 Chambos cost?

3. To practice multiplying by a three-digit number, you or the children could make up problems like these:

A Tyrannosaurus came to Best Burgers. Here's what it ordered.
Find the cost of each item on the order.

	Total
1. 142 Chambos	
2. 400 large colas	
3. 319 Great Greatburgers	

Division

1. One kind of division problem involves sharing food and how much each person would pay. This would usually involve dividing by a one-digit number.

1. You and a friend share a Greatburger with cheese. How much do you each pay?
2. Three people share a large order of french fries. How much does each pay?

2. Another kind of division problem involves finding how many of one kind of thing can be bought for a given price. This would involve dividing by a two-digit number.

1. How many small orders of onion rings can you get for $2?
2. How many large shakes can you get for $4?

Mixed simple practice

You could make up a page of problems that mixed the kinds of practice already suggested. On such a page the children would have to decide how to solve the problem before they did the computations. The children also could make up such problems and you could write them on a duplicating master for the class to use.

Many-step Problems

The problems arising from menus often require more than one step. The many-step problems suggested from the Best Burgers menu are rather straightforward. You may want to make more complicated, realistic problems that would lend themselves to the use of calculators.

1. Give the children an amount to spend, such as $2. Have them buy any 3 items and find the amount of change they would receive. You could vary the amount of money and number of items:

Amount of money to spend	No. of items	Cost	Change
$2.00	3		
$3.00	6		
$1.80	2		

2. Have two children each place an order, find the cost of each order, and then decide how much more one spent than the other. You could also give the orders for about four children and ask the children to find the differences among the costs.

3. Have the children find an average cost from the menu: What is the average cost of a sandwich or of a drink? Or, give the orders for 5 people and ask the questions, What is the average cost per person? Which orders were more than the average and how much more?

4. Give problems that ask about the differences in costs of different amounts of the different sizes of items: Which cost more, 2 large orders of french fries or 3 small orders of french fries? How much more would 4 large chocolate shakes cost than 5 medium chocolate shakes?

You could make up a sheet with many such problems and little reading:

Circle the purchase that costs more and tell how much more it costs.	Difference in cost
4 large shakes or 5 medium shakes	
6 hamburgers or 5 cheeseburgers	

5. Have the children find the quantity of food they could buy for the same cost as another quantity of food. For example, How many Seaburgers could you buy for the price of 6 Great Greatburgers? How many small shakes could you buy for the price of 12 medium shakes?

On many menus there are special dinners you may order instead of ordering a la carte. If you have a menu like that, have the children find the difference in cost in these two ways of ordering.

Graphing Problems

There are many graphing projects that can stem from this one menu. You will probably think of others as you read through this list. And other menus may lend themselves to graphing problems we haven't even mentioned here, so you will need to look over any menu closely to see what graphing problems make sense for it. One idea that might be better done with a different kind of menu is graphing how many of each of different types of foods appear on the menu:

entrees, sandwiches, appetizers, drinks, desserts

Here are our suggestions for the Best Burgers menu.

1. *Individual graphs*

Each child could make a graph of the prices of all the sandwiches or all the drinks on the menu. Using the graphs, questions about the order of these prices could be answered.

2. *Whole-class graphs*

You could have the children in the class vote on their favorite sandwich or drink on the menu, and graph the results, as suggested earlier. Or the children could vote on their *most disliked* sandwich or drink. Or each child could number his or her top three favorite sandwiches in order 1, 2, 3. The children would need to decide how to show and interpret this ranked information.

3. *Whole-grade-level graphs*

The children could collect information about the favorite sandwich or drink or the most disliked sandwich or drink from their whole grade level and graph the results.

4. *Whole-school graphs*

The children could collect favorite sandwich or drink information from each grade level. They could graph the results for each grade level separately and compare favorites for different grades. They could also combine the results for the grade levels and make one graph for the whole school.

5. *Child and adult graphs*

The children could interview a certain number of adults and an equal number of children about their favorite sandwiches or drinks on the menu. They could then make one graph of the adults' responses and one of the children's responses and compare the graphs.

Combination Problems

Many combination problems arise naturally from menus. For example, how many different choices of drinks are there on a menu, or what different combinations of a sandwich and a drink can you choose? Unless the children are familiar with combination problems they should list the possibilities rather than just telling how many possibilities there are. They can organize their lists by making grids (fig. 1).

For most menus you will need to restrict the choices or the listing of all the combinations will become unwieldy for most children. The sample of problems from the Best Burgers menu illustrates ways to restrict the choices. For younger children you may want them to find as many possibilities as they can, without expecting them to find all the possibilities. For children who are more familiar with combination problems, you can remove many of the restrictions. Menus from pizza or ice-cream parlors will provide interesting variety in combination problems.

1. You are ordering one nonbeef sandwich and one soft drink. List the combinations you can order. (The combinations are restricted by limiting the type of sandwich and drink. As a result, there are 12 possible combinations. If there was no restriction on the types of sandwiches and drinks, there would be 247 possible combinations.)

2. You are ordering a sandwich with cheese and a chocolate shake of any of the sizes listed. List the combinations you can order. (There are 18 combinations.)

3. You are ordering a shake. How many choices do you have? (There are 9 choices.)

4. You have 75¢ to spend. You want a sandwich and a milk-type drink. List your possible orders. (Here the child has to consider the price in deciding the 7 possibilities.) If you want to make this a more difficult problem, do not limit the type of drink. Recording the information in a grid or table is one way to organize such a listing (fig. 1).

(The total price of each combination of the two items [sandwich and drink] is given in the table and circled if it is less than or equal to 75¢. There are 21 possibilities.)

Research Problems

Problems that require information above and beyond the menu itself also can be devised. The research involved may mean making a phone call, consulting the encylopedia, interviewing someone at the restaurant itself, using an almanac, and so on. You may feel these types of problems are appropriate for some of your children, but not for others. A child who is willing to take some initiative and dig for information might be very intrigued by such problems. Here are a few that the Best Burgers menu suggested to us, but don't stop with these. See what you can think of.

1. *The cheese-slice problem*

The question here is, How much does a slice of cheese comparable to that used on each sandwich cost when bought by the slice in a grocery store?

2. *The calorie-counting problem*

A child picks what he or she would order at Best Burgers and then determines the total number of calories for that order.

3. *The comparing-restaurants problem*

This problem involves comparing the prices and quantities of featured foods at one restaurant with those of another restaurant. For example, a

Fig. 1

Drinks		Hamburger .35	Cheeseburger .42
Small orange	.25	(60)	(67)
Small cola	.25	(60)	(67)
Medium orange	.30	(65)	(72)
Medium cola	.30	(65)	(72)
Large orange	.40	(75)	82
Large cola	.40	(75)	82
Small chocolate shake	.40	(75)	82
Small vanilla shake	.40	(75)	82
Small strawberry shake	.40	(75)	82
Coffee	.20	(55)	(62)
Milk	.25	(60)	(67)
Hot chocolate	.30	(65)	(72)
Iced tea	.30	(65)	(72)

child could compare a Whopper from Burger Chef and a Big Mac from McDonalds in terms of price, amount of hamburger, size of bun, ingredients on each, and so on. Or the sizes, prices, and actual amounts of Coke sold at two or three drive-in restaurants could be compared.

4. *The How much?/How many? problems*

Problems of this kind have many possibilities. The children could make up their own questions about featured menu items at local restaurants and then investigate several of them. Some possibilities include—

(*a*) How much does it cost the restaurant to make a hamburger? How much more does the restaurant sell a hamburger for?

(*b*) How many hamburgers will one bottle of catsup (the size the restaurant uses in its kitchen) cover?

(*c*) How many hamburgers will 10 kg of hamburger make?

5. *The amount-sold-in-one-time-period problem*

A child could monitor a local restaurant for an hour and keep track of everything sold. This information could be put into a table or graph, and predictions could be made for larger time periods. Or the restaurant might give a child this information for a whole day, and the child could make predictions from it.

6. *The What-else-should-be-on-the-menu? problem*

A child or group of children could interview other children and adults and then make recommendations about what items could be added to the menu and how much might be charged for the individual items.

7. *The best-buy problem*

A child could find out how much meat is in each type of burger on the menu and decide which burger is the best buy. He or she could also find out answers to questions like these:

1. A large shake costs twice as much as a small one. Does it have twice as much in it?

2. A large orange is 15¢ more than a small one. Is it worth it?

3. Is the large order of fries a better buy than two small orders of fries?

Far-Out Problems

We began talking about menus as a source of mathematics problems, but before we finished we had begun to see them as a point of departure for some unexpected destinations. We discovered that menus had possibilities that cut across content areas, just as most everything in the nonschool world cuts across content areas. We were surprised to see how many nonmathematical, but educational, uses of menus we could list. Maybe you and your children will find others.

Language arts

1. Children could devise their own menus, as mundane or imaginative as they wish. Consider, for example, a menu that might have been appropriate for the characters in *Star Wars*.

2. Children could write stories about a favorite kind of food—My Life as a Hamburger, The Adventures of a French Fry.

3. Children might write poems that focus on the smells, tastes, and colors of favorite foods.

4. Children could write radio ads for a restaurant and tape record them.

5. Children could write and videotape television ads for a restaurant.

Art

1. Children could design and draw menus.

2. Children could make papier-mache foods.

3. Children could draw posters advertising the restaurant.

Geography

1. Children could trace the origins of the ingredients in some item on the menu.

2. Children could gather menus from other countries and compare them to local ones.

3. Children could write to relatives to obtain menus from other sections of the United States. They could then compare prices in various sections of the country, or they might examine what items are included in some sections but not in others.

Science

1. Children can investigate how a given food on the menu is produced. For example, How is coffee made?

2. Children can investigate the nutritional makeup of a food on the menu and classify the ingredients by percentages.

History

Children could investigate how and when various foods were discovered or invented. For example, When and where was hamburger first used? How were milk shakes invented? How long has coffee been used? Who invented french fries?

Careers

Children could examine careers connected with restaurants. Parents who work in restaurants could be invited to explain their jobs.

Ecology

Children could look at problems of pollution and waste related to restaurants and make suggestions for improvement.

Afterthoughts

Within the categories or problems in this article we have suggested a variety of ways the Best Burgers menu and other menus can be used. We would like to reiterate some of these ways here and to add some that may have only been hinted at earlier.

1. The children can make up their own problems or you can make up problems.

2. You can designate a general type of problem, and the children can choose their own specifics. For example, you could ask them each to make up and solve ten addition problems about the menu.

3. Two or more menus could be compared in terms of prices or items available.

4. The whole class could use copies of one menu and all answer the same questions about it, or different children could answer different questions about it.

5. Different children could use different menus. In fact, each menu could be seen as an activity card and certain

kinds and levels of questions could be asked about it. The menus could then be organized by you in terms of kind of problem and level of difficulty, thus providing room for much individualization within your class.

6. You and teachers from other grade levels could make up a set of menu cards that could be shared and used across several grade levels. If each of you collected and made up a few such cards you would be saving some time and energy and pooling your ideas. The cards could be used as a kit and could travel from classroom to classroom.

The thoughts of you and the children about menus can make some exciting mathematics.□

The River bank will wake up when the New Standards arrive!

The rat and the mole from "the Wind in the Willows"
Went to math class and took their own pillows.
Their plans were for snoring
Should their lessons grow boring
And their thoughts should go soaring 'mid the clouds and the billows.

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How **Principals** can use the **Arithmetic Teacher**

The school principal is often asked questions that relate to the school's mathematics curriculum and instruction:

Parent: Do you permit students to use calculators during mathematics instruction?

Teacher: How can I make my mathematics lessons more interesting?

Superintendent: The board is thinking of adopting a minimal competency program. What are your thoughts on minimal competencies?

Parent: Why is the metric system being taught?

Teacher: Can you provide me with some techniques for helping me individualize my mathematics instruction?

The *Arithmetic Teacher* can provide a principal with answers to such questions.

As the educational leader in her or his building, the principal's major responsibility is the improvement of the school's instructional program. In spite of the many other administrative functions, instructional leadership and curriculum improvement are the principal's major responsibilities. The *Arithmetic Teacher* includes a wealth of ideas and resources to assist the principal in performing these important tasks.

Articles such as "Calculators in the Classroom" (December 1980) and "Calculators in Testing Situations: Results and Implications from National Assessment" (January 1981) contain information to help the principal answer the first parent's question in reference to calculator use in schools.

Featured articles and three regular departments of the *Arithmetic Teacher* provide the principal with the means to answer the first teacher's question. Helping teachers improve the teaching-learning situation is a goal shared by both the principal and the *Arithmetic Teacher.* Each issue of the journal contains an "IDEAS" section which presents activities that are appropriate for use with students at various levels. Four activity sheets or two posters are reproducible for classroom use. "Let's Do It!" focuses on specific activities with readily available materials and provides a firm basis for teachers needing assistance in planning lessons. "From the File" serves as a teacher exchange or swap-shop of successful teaching tips, projects, games, interesting problems, and lessons—all in card format for easy filing.

The minimal competency movement is affecting mathematics instruction in schools in all areas of our country. What are minimal competencies? Who determines them? How do they affect evaluation of pupil progress? Featured articles on minimal competencies and basic skills appear regularly in the *Arithmetic Teacher* and can provide the principal with an answer to the superintendent's question.

The second parent's question concerning the teaching of the metric system is one of the questions most often asked of teachers, supervisors, and principals. The *Arithmetic Teacher* regularly contains articles on measurement such as "Meaning Metrics: Measure, Mix, Manipulate, and Mold" (March 1981) and "Measurement Activities Using the Metric System" (October 1979) that contain information on the necessity for teaching the metric system of measurement.

Helping teachers to provide for individual differences and learning styles is an important responsibility of the principal. "Do You See What I Hear? A Look at Individual Learning Styles" (November 1979) and "Using Questions to Diagnose and Remediate" (November 1979) are typical of articles related to this subject and can provide the principal with strategies and suggestions to answer the second teacher's question.

"Reviewing and Viewing," which contains reviews of instructional materials and books for teachers and pupils, is another regular department of the *Arithmetic Teacher.* This column can provide the principal with the necessary background information for making important decisions when books and materials are to be requisitioned. The monthly reviews can be shared with the school's librarian to aid her or him in selecting books on mathematics subjects for students and reference materials for teachers.

Each issue of the *Arithmetic Teacher* contains "Professional Dates" of national, regional, and local meetings of mathematics educators. The principal who shares this information with her or his faculty encourages the professional development of teachers.

A principal must be informed on current trends in mathematics education. "What's Going On . . ." is a regular page of the *Arithmetic Teacher* containing information on new projects and programs, effective curricular procedures, and legislative actions.

The *Arithmetic Teacher* is one of the most valuable features of membership in the NCTM. Principals interested in improving mathematics instruction and keeping abreast in mathematics education need an *Arithmetic Teacher* in their schools. ■

By David E. Williams
Member of the Editorial Panel

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EVALUATION STANDARDS

OVERVIEW

This section presents fourteen evaluation standards, categorized by focus: General Assessment, Student Assessment, and Program Evaluation.

General Assessment

1. ***Alignment***
2. ***Multiple Sources of Information***
3. ***Appropriate Assessment Methods and Uses***

Student Assessment

4. ***Mathematical Power***
5. ***Problem Solving***
6. ***Communication***
7. ***Reasoning***
8. ***Mathematical Concepts***
9. ***Mathematical Procedures***
10. ***Mathematical Disposition***

Program Evaluation

11. ***Indicators for Program Evaluation***
12. ***Curriculum and Instructional Resources***
13. ***Instruction***
14. ***Evaluation Team***

Evaluation and Change

A common response to the challenge of the *Standards* is, "Yes, but who will change the tests?" Although pragmatic, this question shifts responsibility for change away from the individual to some unnamed higher authority. More productive—and more likely to make the vision embodied in the *Standards* a reality—are such responses as, "In what ways does the curriculum need to be changed?" "How best can these changes be made?" "How will we know when we have reached the *Standards*?" It is in the answers to these questions that the role of evaluation emerges as a critical component of reform. Evaluation is a tool for implementing the *Standards* and effecting change systematically. The main purpose of evaluation, as described in these standards, is to help teachers better understand what students know and make meaningful instructional decisions. The focus is on what happens in the classroom as students and teachers interact. Therefore, these evaluation standards call for changes beyond the mere modification of tests.

Yes, tests also need to change. They must change because a curriculum that fulfills the *Standards* will differ significantly, in both content and instruction, from most existing curricula. Many existing tests cannot measure the student outcomes identified in the *Standards*, for example, having K–4 students use many computation techniques and having 9–12 students make connections among mathematical topics. As the curriculum changes, so must the tests. Tests also must change because they are one way of communicating what is important for students to know. The tested curriculum can strongly influence what students are taught.

♦ ♦ ♦ ♦ ♦ ♦ ♦ ♦

In this way tests can effect change. Finally, existing tests must change because they are based on different views of what knowing and learning mathematics means. Knowing mathematics by doing mathematics in a technological world differs from developing a sequence of skills or objectives when calculators and computers did not exist and when mathematical applications were primarily confined to the physical sciences and commerce.

Evaluation is fundamental to the process of making the *Standards* a reality. Just as the Curriculum Standards propose changes in K–12 content and instruction, the Evaluation Standards propose changes in the processes and methods by which information is collected. These changes, developed in light of current knowledge about evaluation, are intended to increase and improve the gathering of relevant, useful information. Assessment and program-evaluation practices must change along with the curriculum. The Evaluation Standards propose that—

- student assessment be integral to instruction;
- multiple means of assessment methods be used;
- all aspects of mathematical knowledge and its connections be assessed; and
- instruction and curriculum be considered equally in judging the quality of a program.

Other Issues

When programs are evaluated and students are assessed, the information collected must be aggregated to draw meaning from what was observed or measured. The aggregated information is reported, used to assign grades, or used as an indication of the quality of the program. Although the process of aggregating information, scoring, and reporting is not discussed in detail in these Evaluation Standards, these issues are important and should be addressed. Because the Evaluation Standards call for multiple means of assessment, they imply that a variety of scoring schemes are to be used. Such schemes can include ratings of students' work that are based on its overall quality or on the inclusion of specific features or parts. Record keeping can be writing notes on cards or on a computer or maintaining a portfolio of the students' work. Whatever the scheme, the results should constitute an accurate and thorough indication of the mathematics that students know. Merely adding scores on written tests will not give a full picture of what students know. The challenge for teachers is to try different ways of grading, scoring, and reporting to determine the best ways to describe students' knowledge of mathematics as indicated in these *Standards*.

Format of the Evaluation Standards

The Evaluation Standards are organized in three sections: General Assessment, Student Assessment, and Program Evaluation. The general-assessment standards discuss principles relevant to any form of assessment and program evaluation. The student-assessment standards consider aspects of mathematical knowledge that should be assessed, as derived from the Curriculum Standards. The program-evaluation standards examine the assessment of the extent to which a mathematics program is consistent with the *Standards*.

The format of the Evaluation Standards differs from that of the Curriculum Standards. Each evaluation standard begins with a statement about

the topics or concepts that an assessment or program evaluation should address, followed by a list of indicators describing what can be observed. Indicators are used to identify outcomes because they denote a measure placed in some context. In this situation, the Curriculum Standards serve as the criteria against which to compare the evidence of what students can do.

Emphases of the Evaluation Standards

The fourteen Evaluation Standards emphasize aspects of assessment and program evaluation that depart from current practice. The following aspects are to receive increased and decreased attention:

Increased Attention	Decreased Attention
♦ Assessing what students know and how they think about mathematics	♦ Assessing what students do not know
♦ Having assessment be an integral part of teaching	♦ Having assessment be simply counting correct answers on tests for the sole purpose of assigning grades
♦ Focusing on a broad range of mathematical tasks and taking a holistic view of mathematics	♦ Focusing on a large number of specific and isolated skills organized by a content-behavior matrix
♦ Developing problem situations that require the applications of a number of mathematical ideas	♦ Using exercises or word problems requiring only one or two skills
♦ Using multiple assessment techniques, including written, oral, and demonstration formats	♦ Using only written tests
♦ Using calculators, computers, and manipulatives in assessment	♦ Excluding calculators, computers, and manipulatives from the assessment process
♦ Evaluating the program by systematically collecting information on outcomes, curriculum, and instruction	♦ Evaluating the program only on the basis of test scores
♦ Using standardized achievement tests as only one of many indicators of program outcomes	♦ Using standardized achievement tests as the only indicator of program outcomes

Some Evaluation Ideas

Nancy Hord

Editor's Note: The following short-hand expressions represent the myriad ways in which math concepts can be taught and evaluated in ways which will help develop rich conceptions among young learners. These are very consistent with the ways in which the Standards appeal for the development and evaluation of richness in the curriculum.

Translations Between Modes of Representation

Modalities: P = Pictures; WS = Written Symbols; M = Manipulatives; SS = Spoken Symbols; RW = Real World Situations

Concept: subtraction of whole numbers less than 10

RW→M: Story: You had six sticks of gum and you gave two away to your friend. Use these rods to show the story and find out how many sticks of gum you have left.

RW→P: (tell gum story) Draw a picture of this story and find out how many sticks of gum you have left.

RW→WS: (tell gum story) Use numerals to write the story (6-2 = 4)

RW→SS: (tell gum story) Tell in your own words what we did in this problem (6 sticks of gum minus 2 sticks of gum leaves 4 sticks of gum)

M→RW: Here are 8 beads. We take away 5 beads and have how many left? Make up a real story using these.

M→P: (use beads again) Draw a picture of what we did with the beads.

M→WS: (use beads again) Show this on the board with written numbers (8-5 = 3)

M→SS: (use beads again) Tell me in your own words what we did with the beads. (We had 8 beads, then we took away 5 beads and we had 3 left.)

P→M: In this picture, we see a group of 6 trucks. Four of the trucks are driving away (enclose 4 in a ring). How many trucks are left? Use these rods to find out how many trucks are left.

P→WS: (same truck story in pictures) Write this problem with numerals. (6-4 = 2)

P→SS: Here is a picture showing a group of trucks. Some of the trucks are driving away and some are left. Tell how many are in the big group of trucks, how many drive away and how many are left (Six trucks take away four trucks leaves two trucks.)

P→RW: Tell a story about the trucks in this picture.

WS→SS: Here is a subtraction problem (write 8-2 =____ (write on the board). Describe this problem in your own words.

WS→P: (same written problem) Draw a picture of this problem and find the answer.

WS→M: (same written problem) Use these sticks to show this problem and to find the answer.

WS→RW: (same written problem) Make up a story using this problem. Can you find out how many are left?

SS→P: Suppose you have 7 valentines and you give away 3 of them. Draw a picture to find out how many are left.

SS→M: (some valentine description) Use these little paper hearts to find out how many are left.

SS→WS: (some valentine description) Show this with written numerals and write how many are left.

SS→RW: Tell a story about valentines which relate to this number sentence: 7-3 =____.

Concept of One-Half

Liz Hiller

I. Given a display of manipulatives (rods) showing one half -
   1) tell me a story using the manipulatives

2) show me a picture using one half
3) what is the numeral which corresponds with or describes the manipulatives
4) write the fraction of the display.

II. As I read my story (Today at lunch I had a sandwich, my friend forgot to bring hers. I gave her half of my sandwich.)...
1) use the rods to show me my sandwich and how much I gave my friend
2) draw a picture of my sandwich and how much I gave my friend
3) tell me the number sentence which goes with my story how I gave her one half of my sandwich
4) write the number (fraction) of the amount I gave my friend.

III. Look at my picture (a half of a pie) and...
1) tell me a story about what happened
2) show me with manipulatives the same amount
3) tell me how much pie is left
4) write the number of the amount of pie left

IV. When I say a fractional number (one-half) I want you to...
1) show me that same number with your rods.
2) tell me when you have used that same number
3) write that number down
4) draw me a picture of something in that amount.

V. I'm going to write a fractional number (one-half) on the board. I want you to...
1) show me that number by folding your paper
2) tell me when your mom would ever use that number
3) draw me a shape which matches or represents that number
4) say that number to me.

Liz and Nancy were graduate students at Tech in the spring of 1989 when they developed these ideas.

Educational Materials

(continued from inside back cove.)

Multicultural Mathematics Materials, *by Marina C. Krause.* Games and activities from around the world to enhance individual students' ethnic identity and enhance the mathematics lesson as well. Introduces children to the ethnic heritage of others. Large-size reproducible activity pages; grades 1-8.
1983 80 pp. ISBN 0-87353-206-6 #327E1 $6.00

I Can Count the Petals of a Flower, 2d rev. ed., *by John and Stacey Wahl.* A fascinating, full-color hardback picture book for teaching young children to count through beautiful photographs of flowers, plus such mathematical concepts as events and odds, primes and composites, and factors. See "Posters."
1985 36 pp. ISBN 0-87353-224-4 #358E1 $8.00

Ideas from the Arithmetic Teacher: Grades 1-4 Primary, *compiled by George Immerzeel and Melvin L. Thomas.* Second in a series of "IDEAS" from the *Arithmetic Teacher;* conveniently grouped by topic, such as problem solving, computation, numeration, measurement, and geometry.
1982 120 pp. ISBN 0-87353-189-2 #279E1 $8.00

Multicultural Mathematics Posters and Activities. 18 attractive posters that prove that mathematics spans centuries and cultures and is useful and fun. Originally produced by the Seattle Public Schools for the middle school. The materials emphasize problem solving, focus on interdisciplinary applications of mathematics, and encourage minority students to pursue mathematics beyond minimum requirements. Includes a 64-page activity book for the teacher.
1984 ISBN 0-87353-213-9 #330E1 $9.00

New Directions for Elementary School Mathematics. (1989 Yearbook). Gets children off to the right start in grades K-6, when they are just setting out on their educational voyage of discovery. Explores areas of profound change in elementary school mathematics, such as problem solving, communication, children's reasoning processes, and how they develop mathematical ideas. Contains practical suggestions for teachers and curriculum developers; descriptions of classroom investigations of measurement, fractions, calculators, statistics, and more; and samples of children's work that reinforce the yearbook's active approaches to learning.
1989 248 pp. ISBN 0-87353-272-4 #400E1 $18.00

Contacts for Local Math Groups

Elementary teachers are welcomed!

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All Councils have meetings once or twice each school year at which there are presentations and workshops for elementary school mathematics.

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Calculators (AT, February 1987). #388E1. $5.00

Early Childhood Mathematics (AT, February 1988). #385E1. $5.00

Manipulatives (AT, February 1986). #354E1. $5.00

Number Sense (AT, February 1989). #408E1. $5.00

Curriculum and Evaluation Standards for School Mathematics. In 1986, NCTM's Board of Directors established the Commission on Standards for School Mathematics to create a coherent vision of what constitutes mathematical literacy and to create a set of standards to guide the revision of the school mathematics curriculum in the next decade. This document is the response to that challenge.

The *Standards* envisions what the curriculum should include in terms of content priority and emphasis. The 54 standards are presented in four groups—one set each for grades K-4, 5-8, and 9-12 and one set for evaluating mathematics programs and student achievement.

The current crisis in the teaching and learning of mathematics will generate many proposals for solving curricular problems and answering evaluation questions. The standards in this benchmark publication are the definitive criteria against which all such proposals can be measured and judged.

Quantity discounts: 10-24 copies—20%; 25-99 copies—30%; 100 or more copies—40%; 1,000 or more copies—$12.50 ea. Copies purchased on the special quantity discount will be shipped to one address only. Copies are nonreturnable and nonrefundable.

1989 258 pp. ISBN 0-87353-273-2 #398E1 $25.00

Developing Graph Comprehension: Elementary and Middle School Activities, *by Frances Curcio*, is an 8.5"x11" perforated collection of activities that teach students how to interpret, construct, and write about traditional graphing formats, such as bar, circle, picture, and line graphs, and new graphing techniques, such as box, line, and stem-and-leaf plots. Contains 23 activities for classroom use as well as instructions for making reusable teaching aids, graph-paper masters, and data collection sheets. Discusses what graphs are and how they are used; steps in collecting, organizing, and analyzing data; and constructing and writing about graphs.

1989 85 pp. ISBN 0-87353-277-5 #403E1 $11.50

How to Evaluate Your Mathematics Program. A tool for assessing program effectiveness and setting directions for change. Developed by teachers, principals, and supervisors from elementary, secondary, and higher education.

1981 24 pp. ISBN 0-87353-183-3 #301E1 $3.00

Double-Column Addition: A Teacher Uses Piaget's Theory, *by Constance Kamii*. A 20-minute videotape that captures the spontaneous reponses and thought processes of second-grade children as they solve double-column addition problems. Segments recorded at two times during the school year show how double-column addition is invented and how social interaction is important in the development of this ability. The teaching process emphasizes discussion, invention, and the encouragement of critical thinking, rather than the traditional rule-based approach. A good catalyst for discussion.

1989 VHS only. #417E1 $55.00

"Early Childhood Mathematics." Because early childhood is a critical period for the development of mathematical ideas and concepts, a special issue of the *AT* has been devoted to this important time. Contains articles on such topics as the early development of number relations, microcomputers, word problems, and place value. Special sections offer suggestions for hands-on activities, some with calculators. Evaluation and related instructional issues are considered.

Arithmetic Teacher, Feb. 1988 #383E1 $5.00

Games and Puzzles for Elementary and Middle School Mathematics: Readings from the Arithmetic Teacher, *edited by Seaton E. Smith, Jr. and Carl A. Backman*. More than 100 articles on the use of games and puzzles to capture student interest and imagination.

1975 280 pp. ISBN 0-87353-054-3 #61E1 $12.00

Geometry for Grades K-8, *edited by Jane M. Hill*. A compilation of articles from the *Arithmetic Teacher*, principally from the past 10 years. The focus is on teaching ideas and activities arranged under the headings of geometric shapes, blocks, patterns and transformations, geometry in unusual ways, and spatial abilities. The hands-on approach is appropriate to the study of informal geometry.

1987 173 pp. ISBN 0-87353-237-6 #365E1 $11.00

How to Choose and Create Good Problems for Primary Children, *by Doyal Nelson and Joan Worth*. With problem solving the focus of school mathematics in the 1980s, this is a booklet for the decade. Guides the teacher in drawing very young children into the problem-solving way of thinking; many activities. First appeared as a chapter in the 37th Yearbook.

1983 40 pp. ISBN 0-87353-205-8 #324E1 $8.00

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(continued on page 47)

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